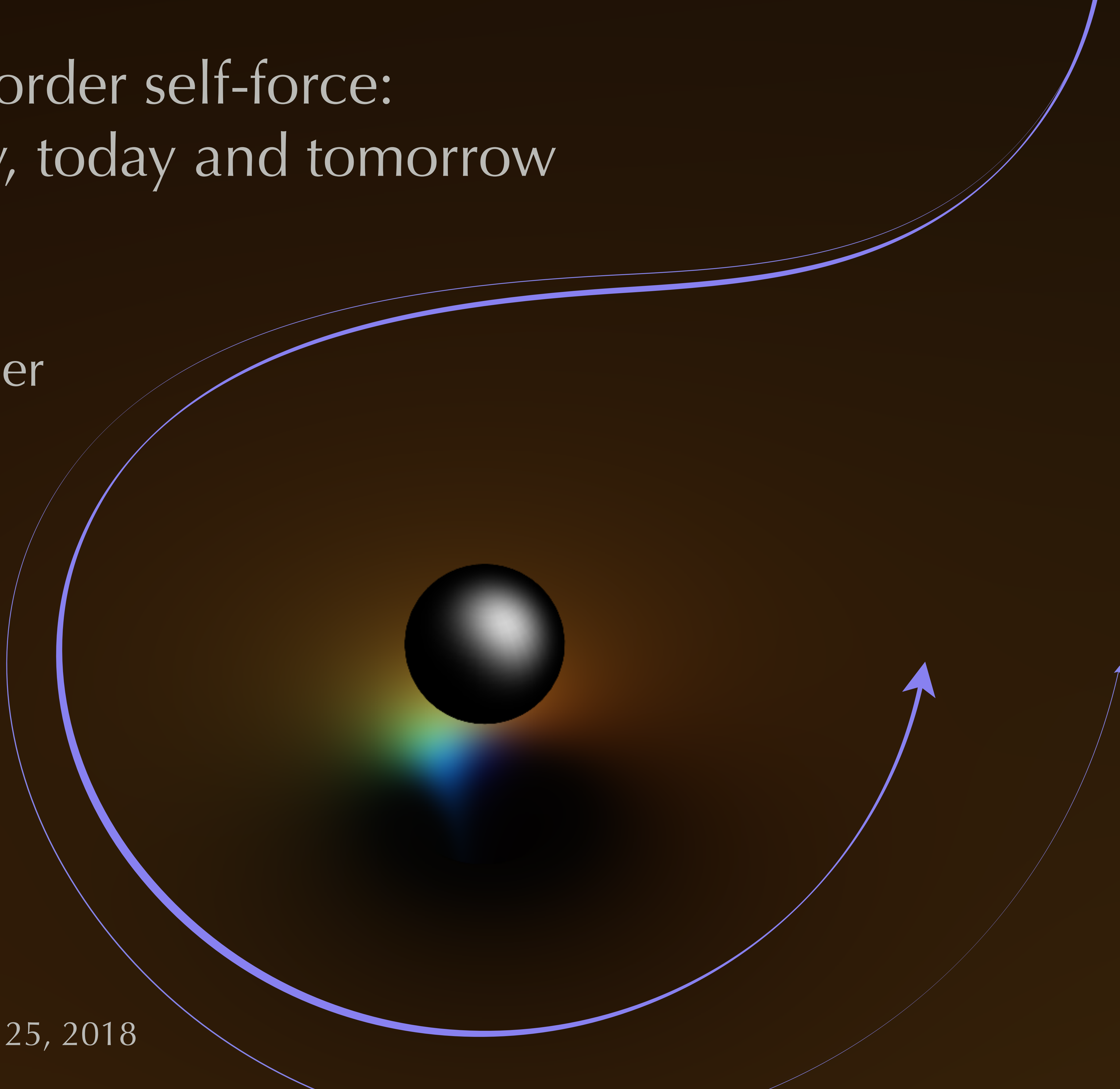
The first-order self-force:
Yesterday, today and tomorrow

Seth Hopper

Earlham
COLLEGE



Capra 21 - June 25, 2018

Outline

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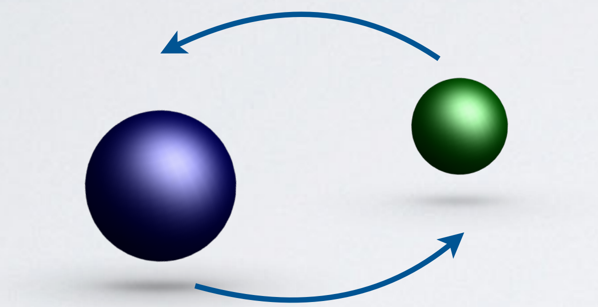
Why we're here



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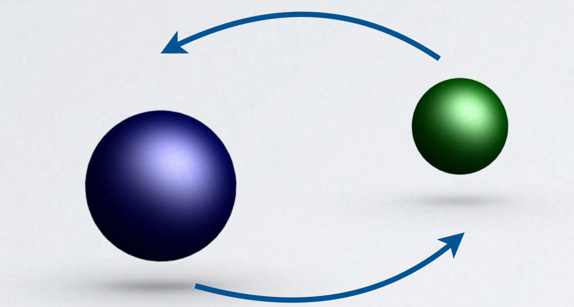
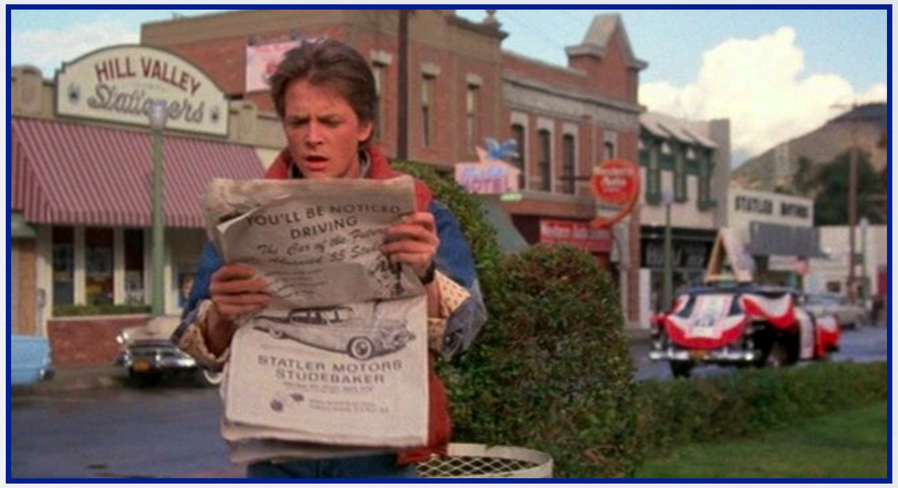


Yesterday



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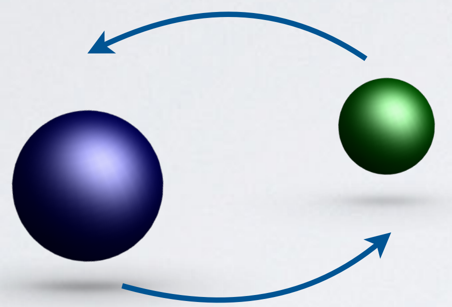
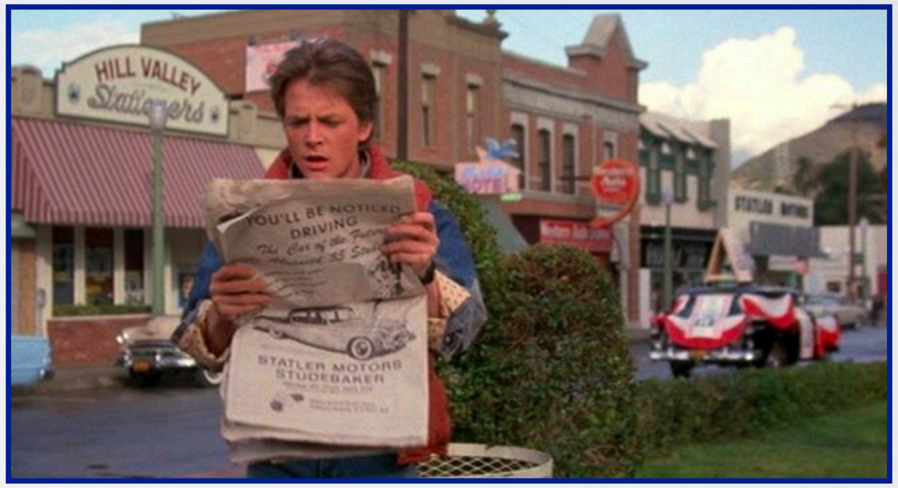


Yesterday



Today

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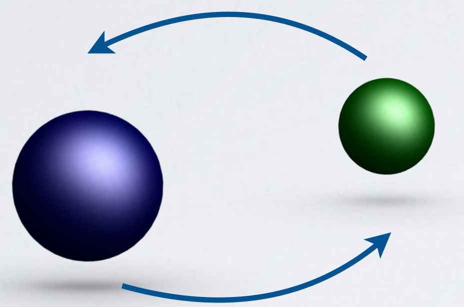
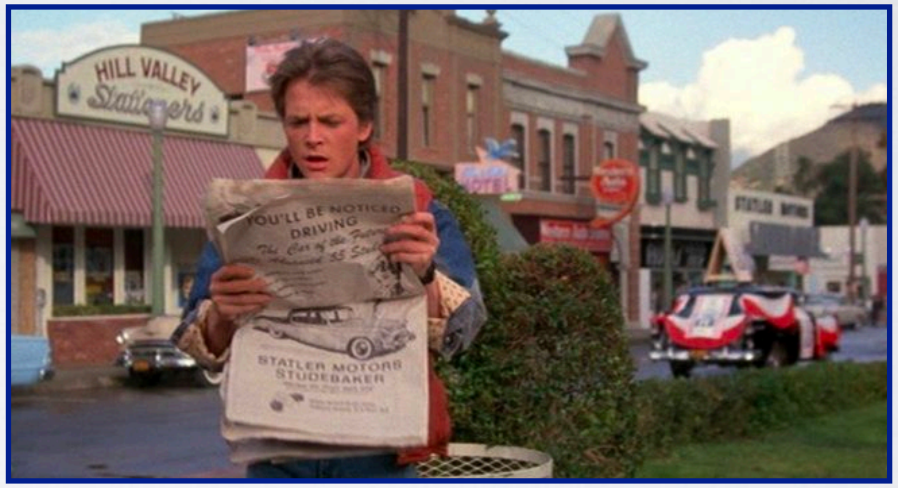


Yesterday



Today

Tomorrow

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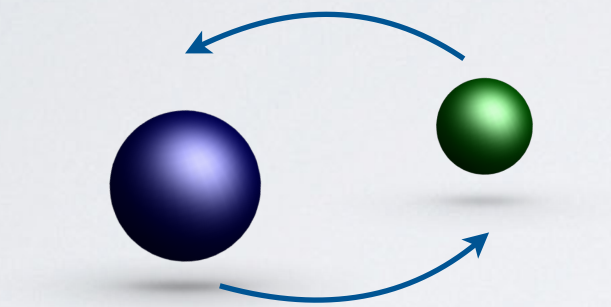
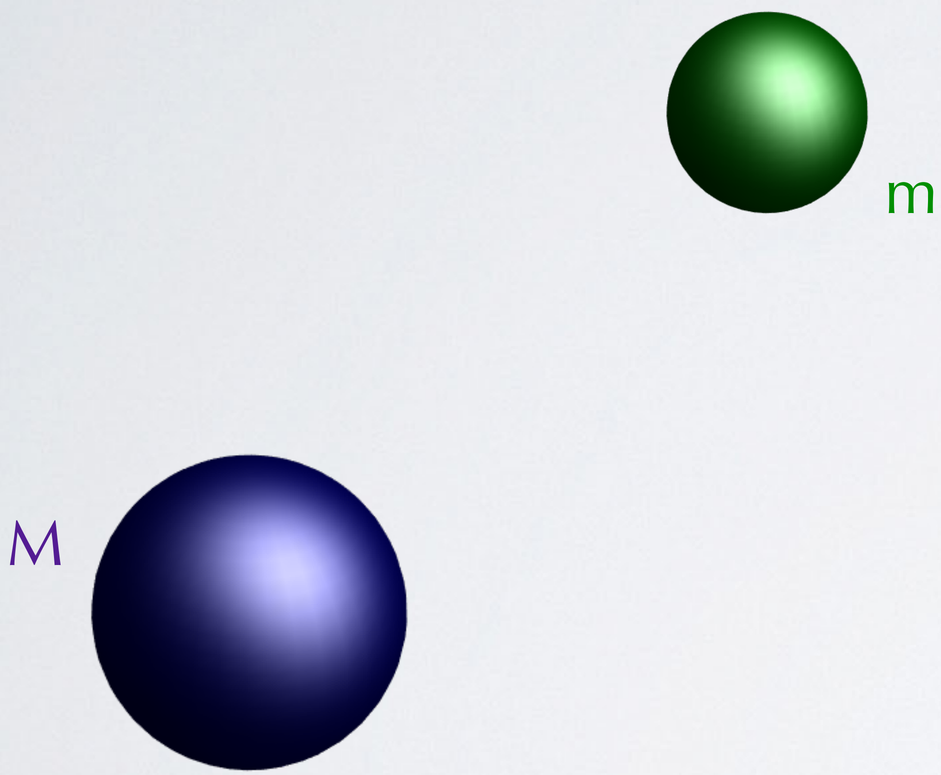


Yesterday

Today

Tomorrow

The two-body problem is inherently interesting



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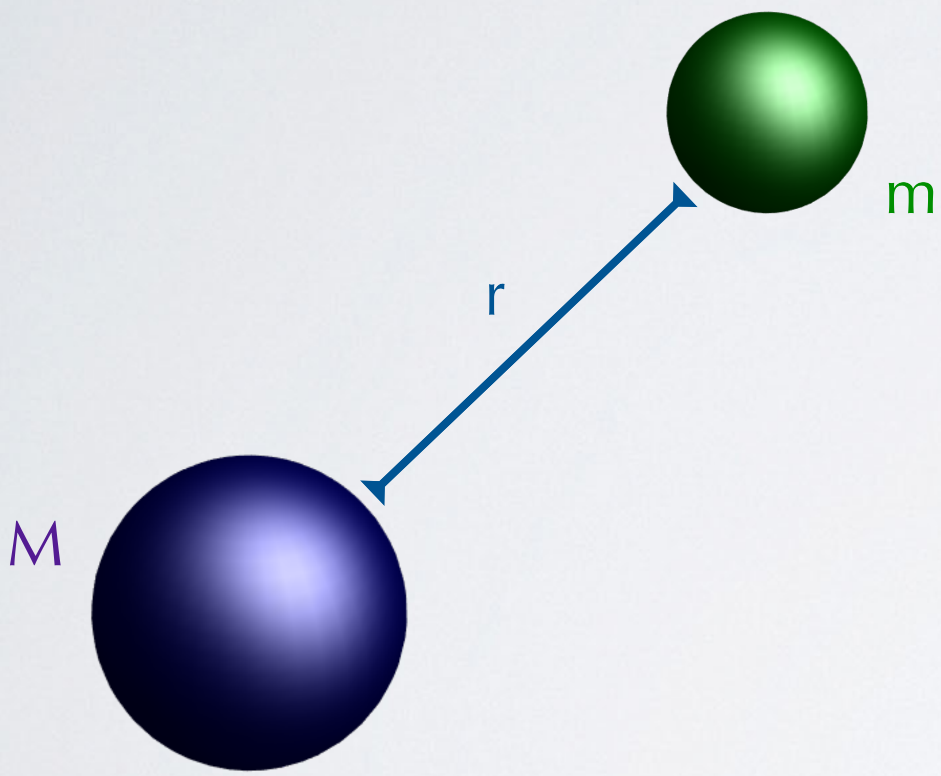

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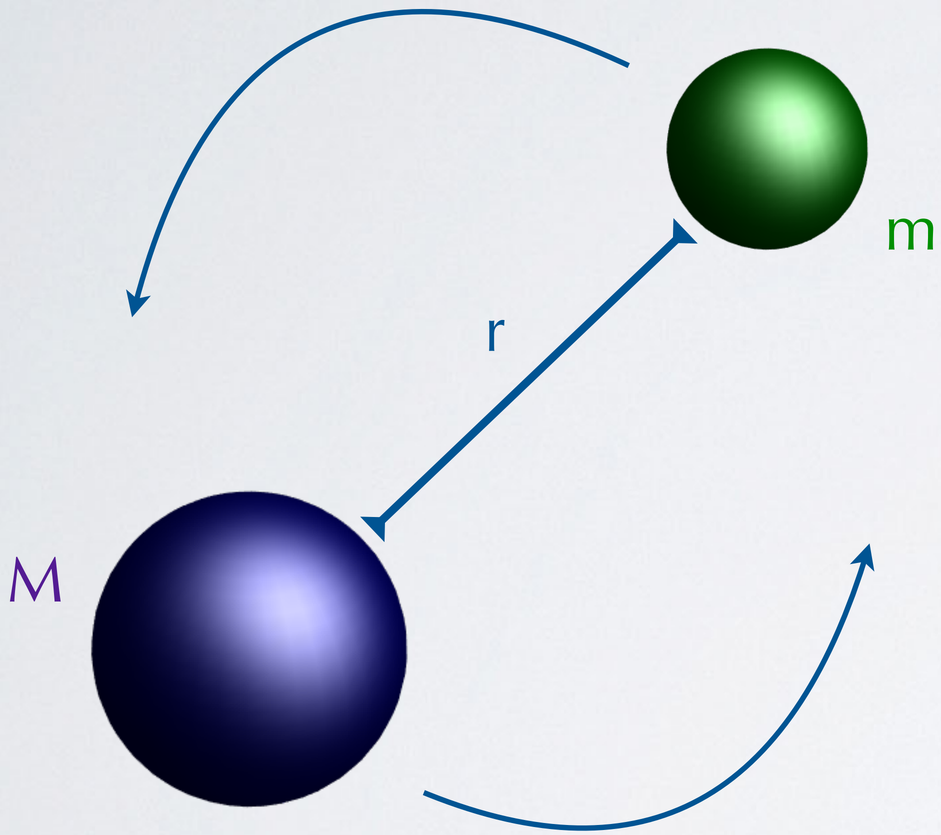

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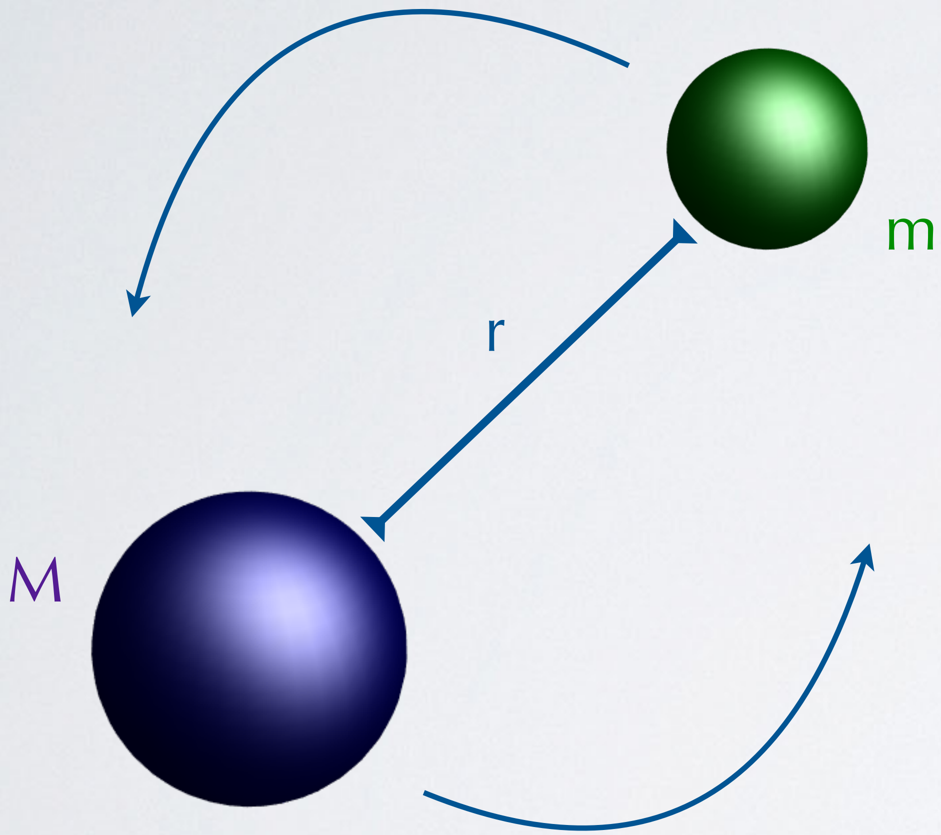

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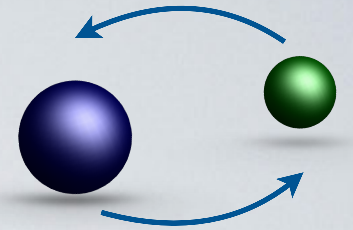

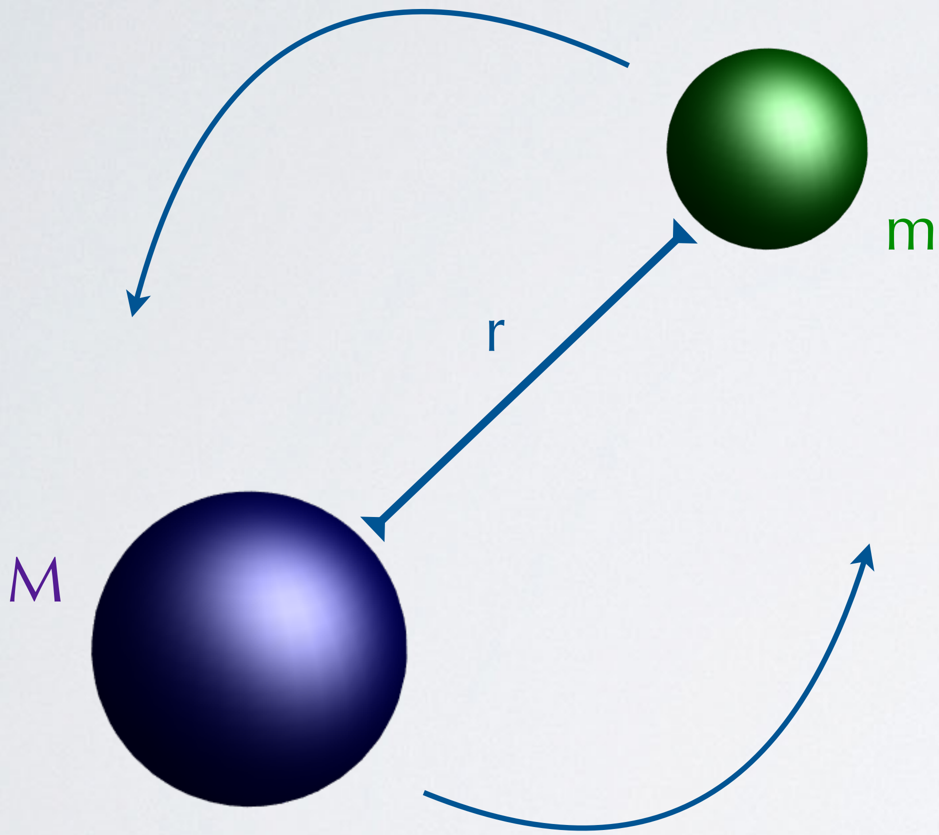

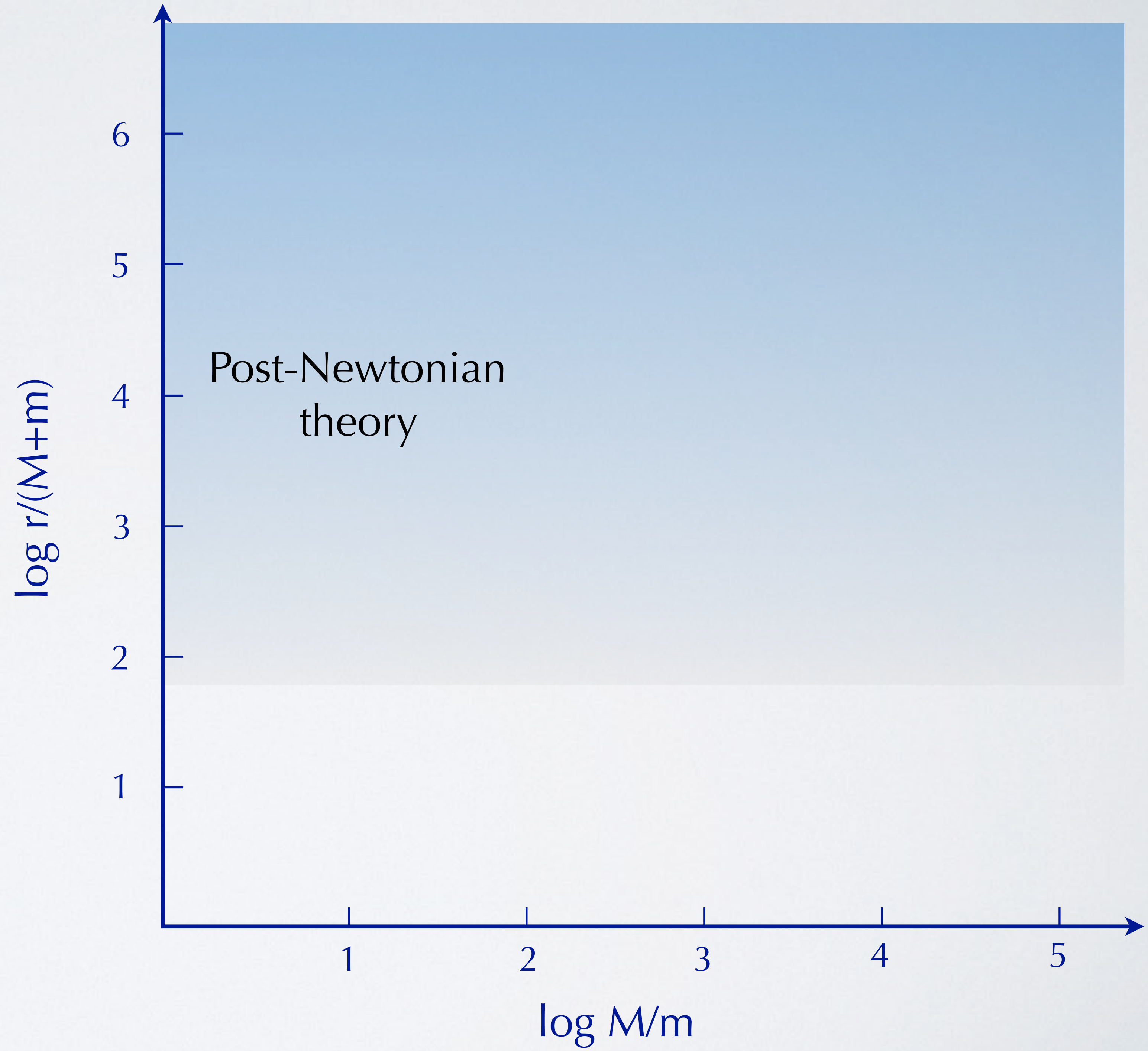

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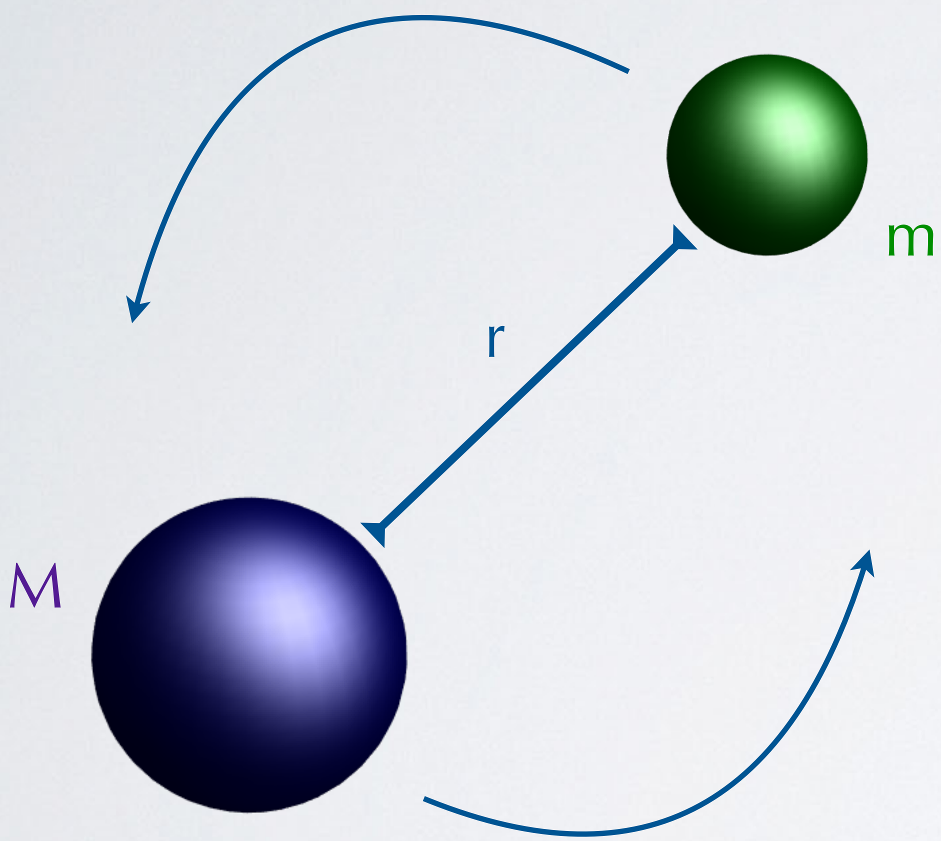

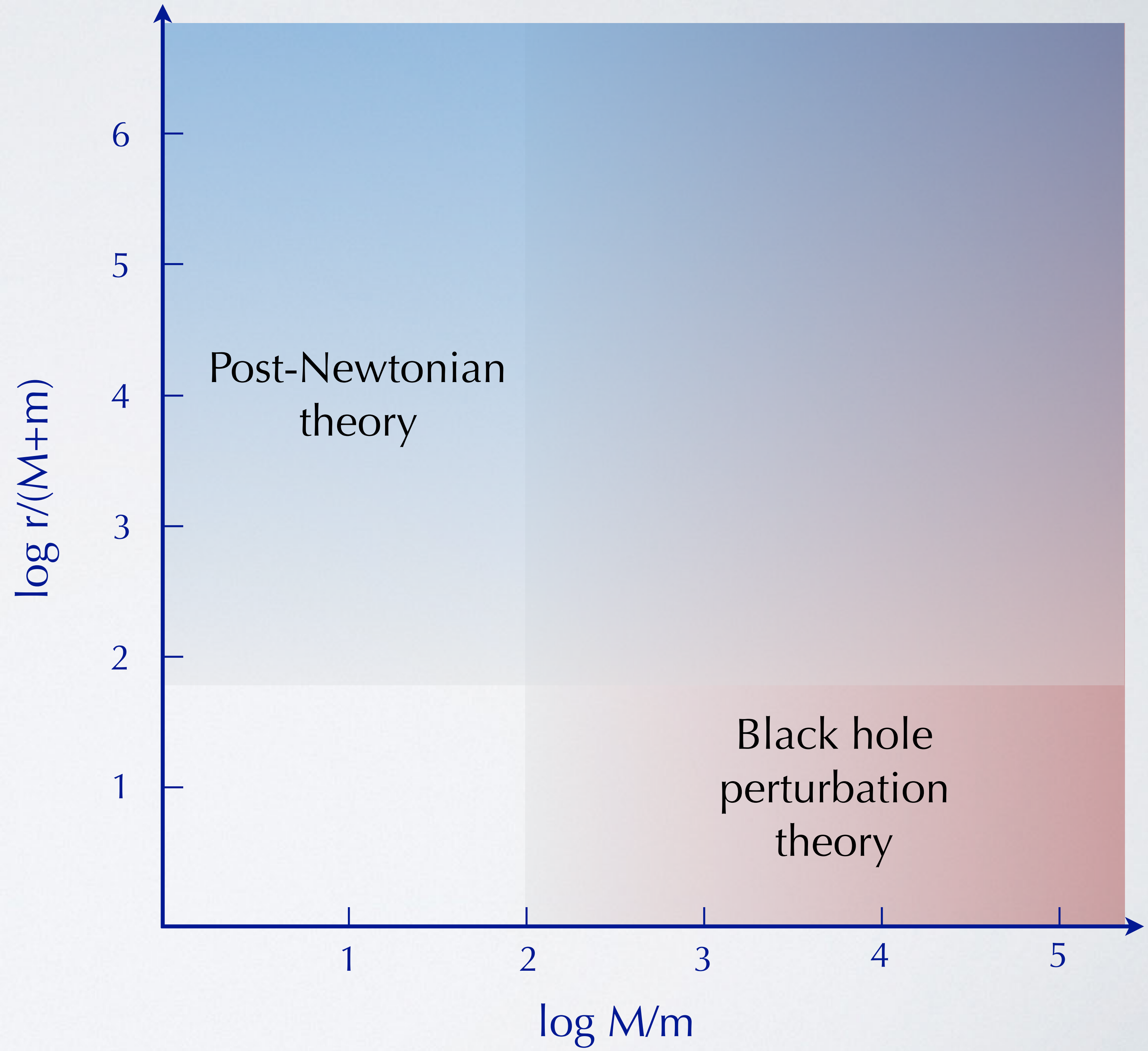

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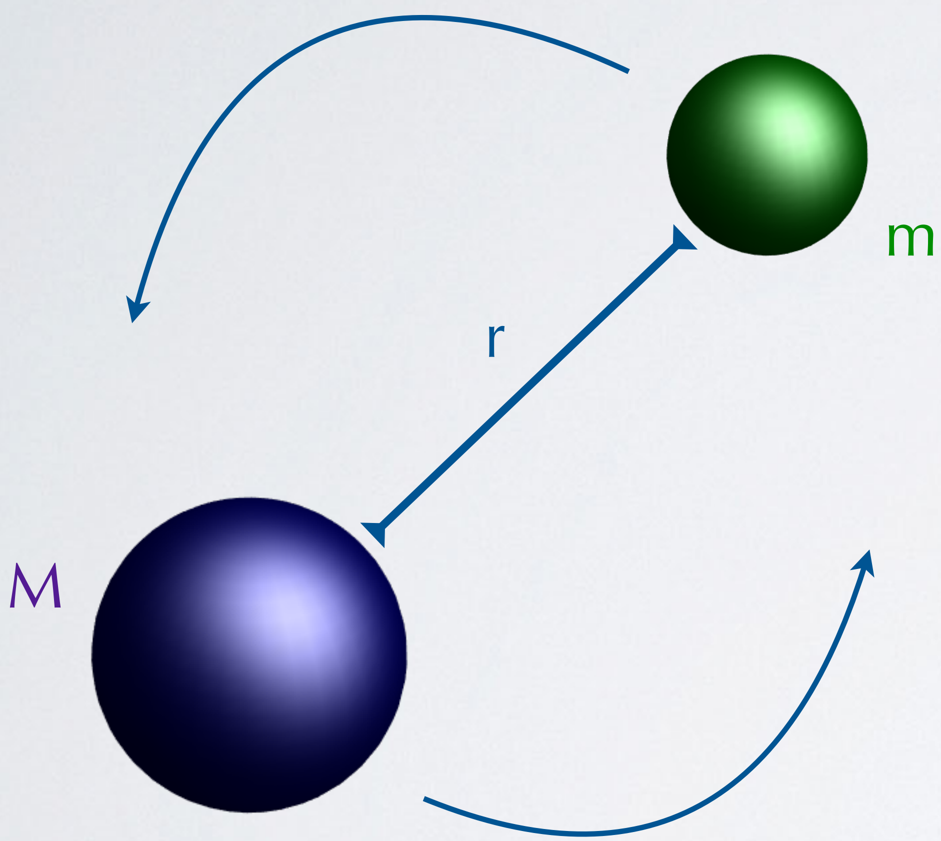

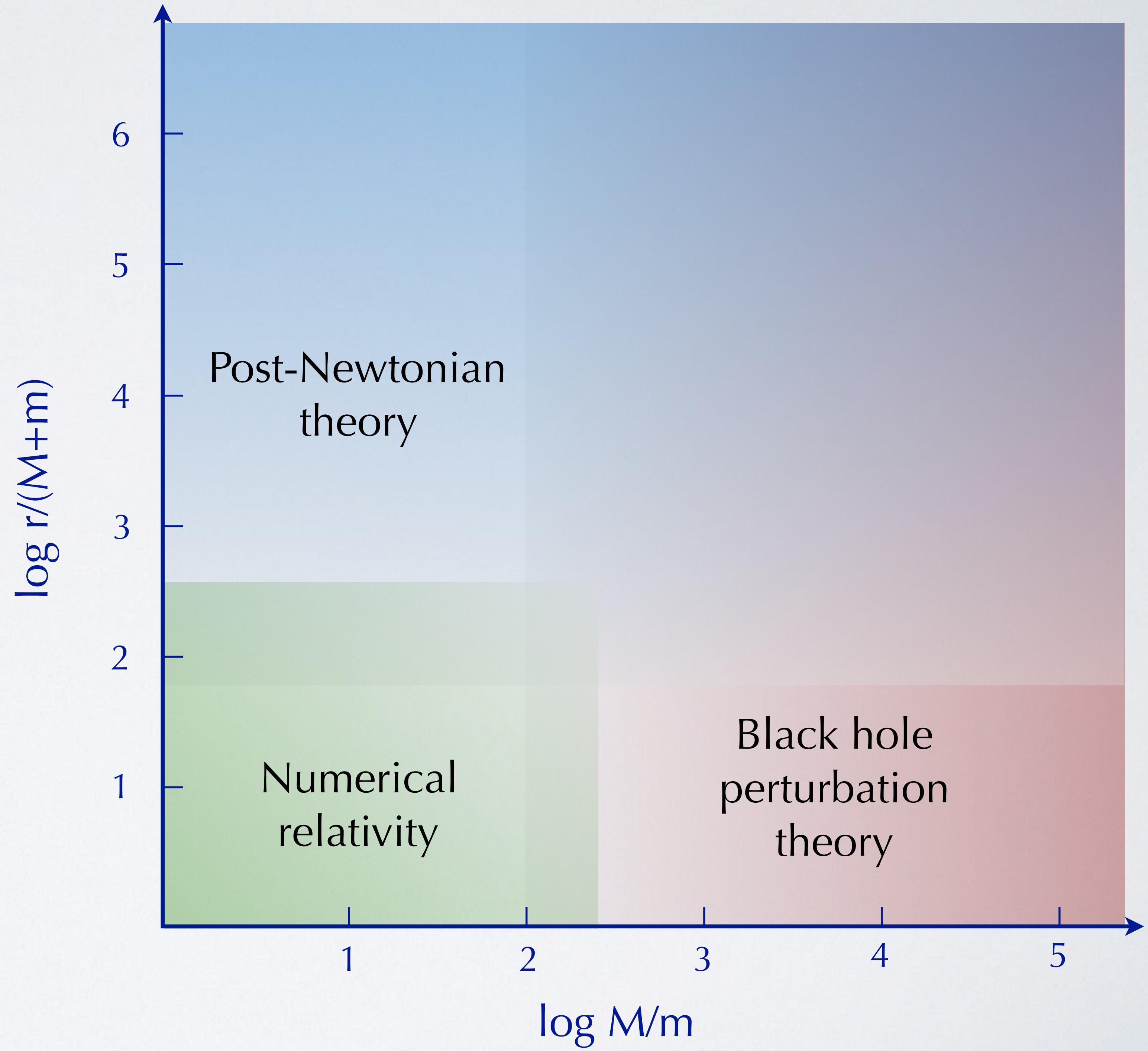

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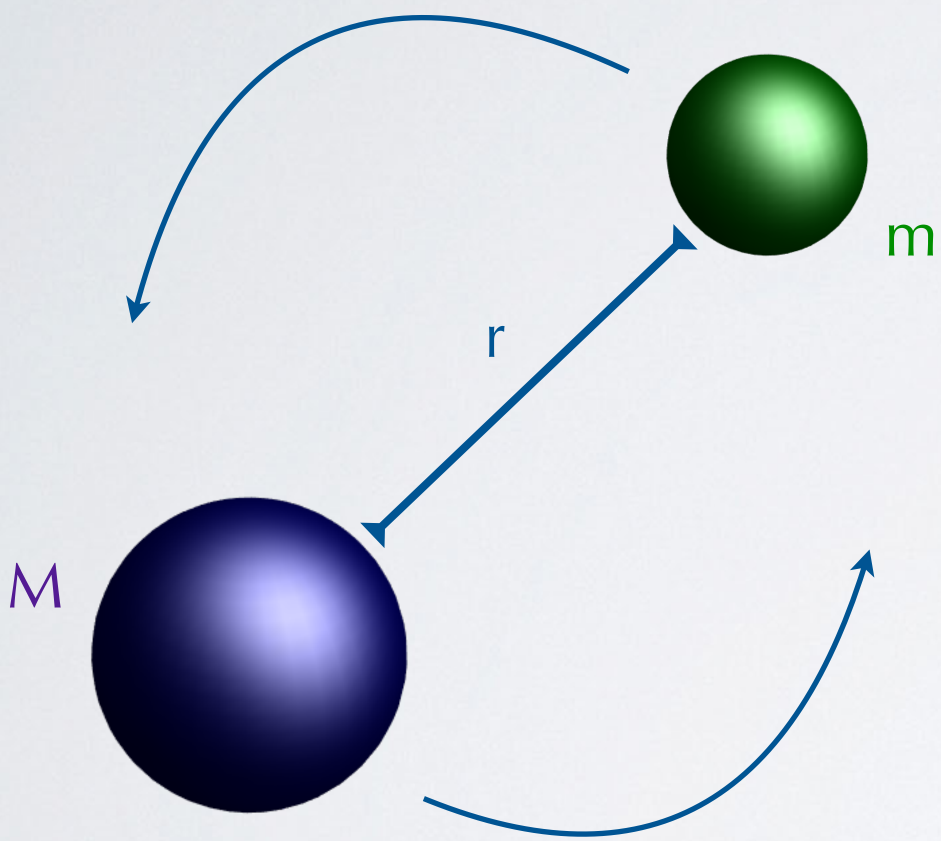

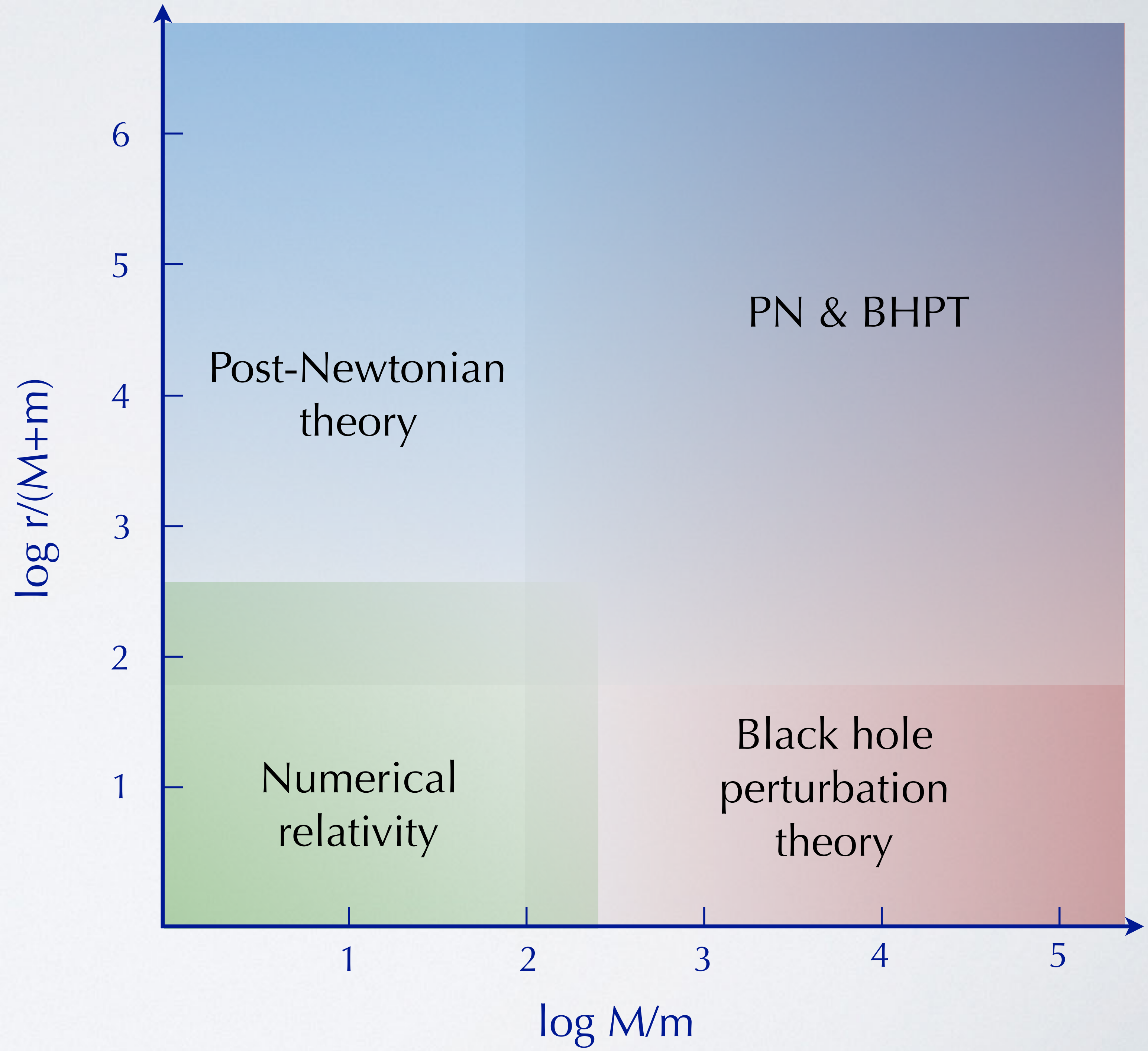

Different techniques provide different parts of the waveform

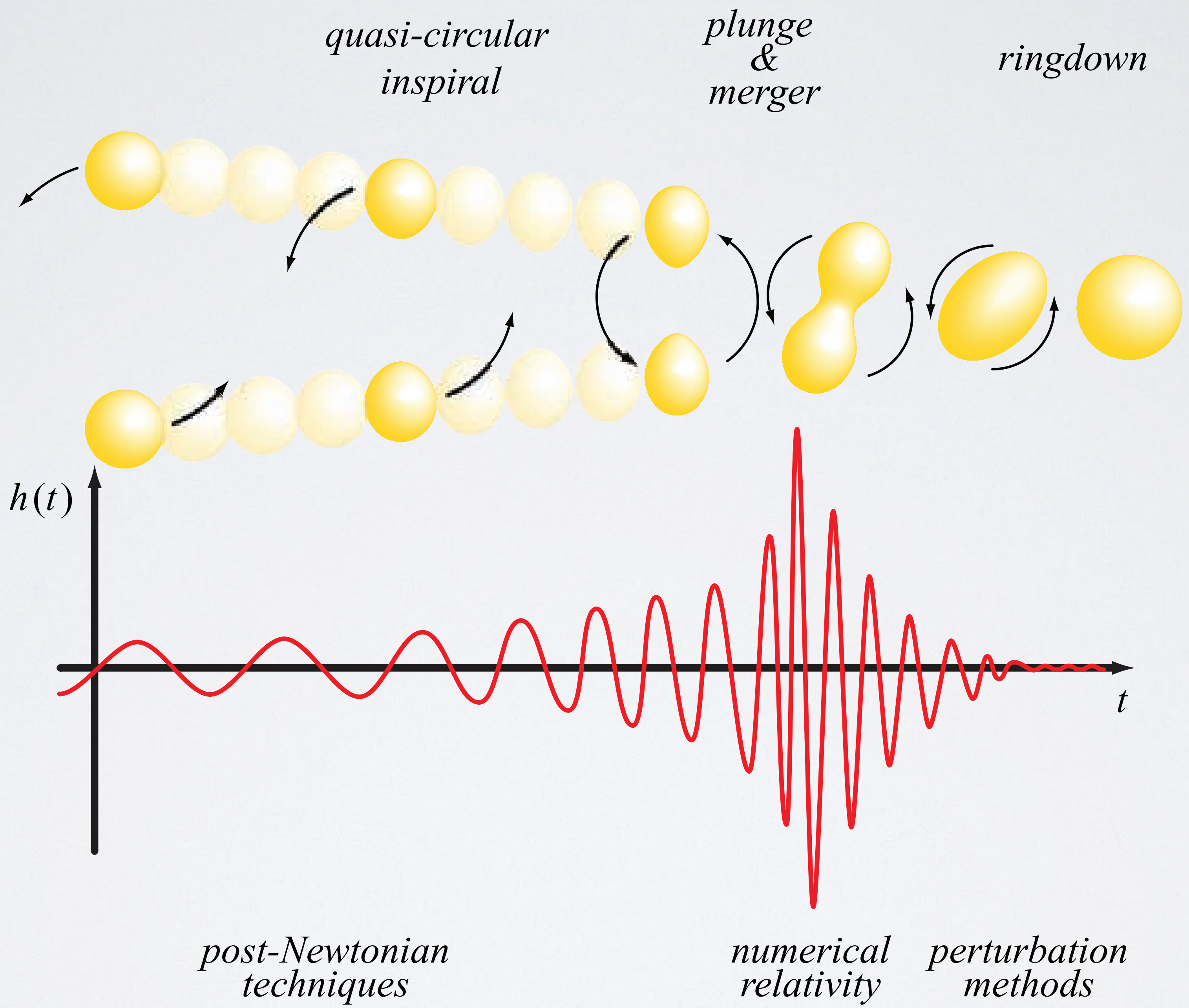

Fig: Baumgarte & Shapiro

We are also motivated by gravitational wave astronomy

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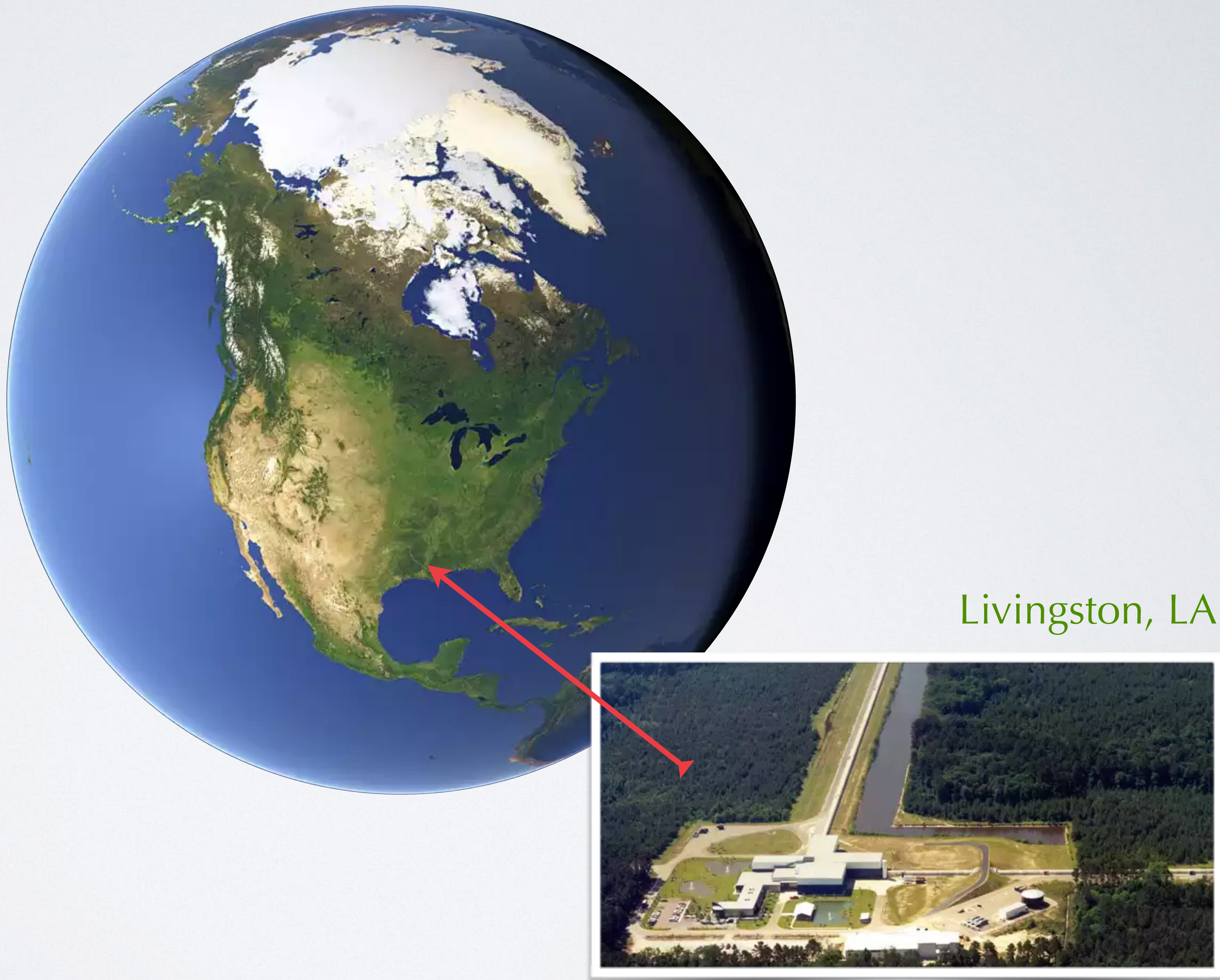

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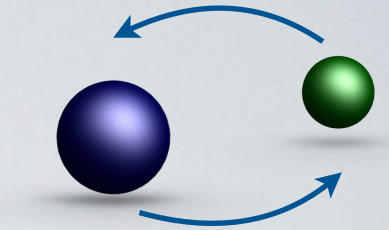

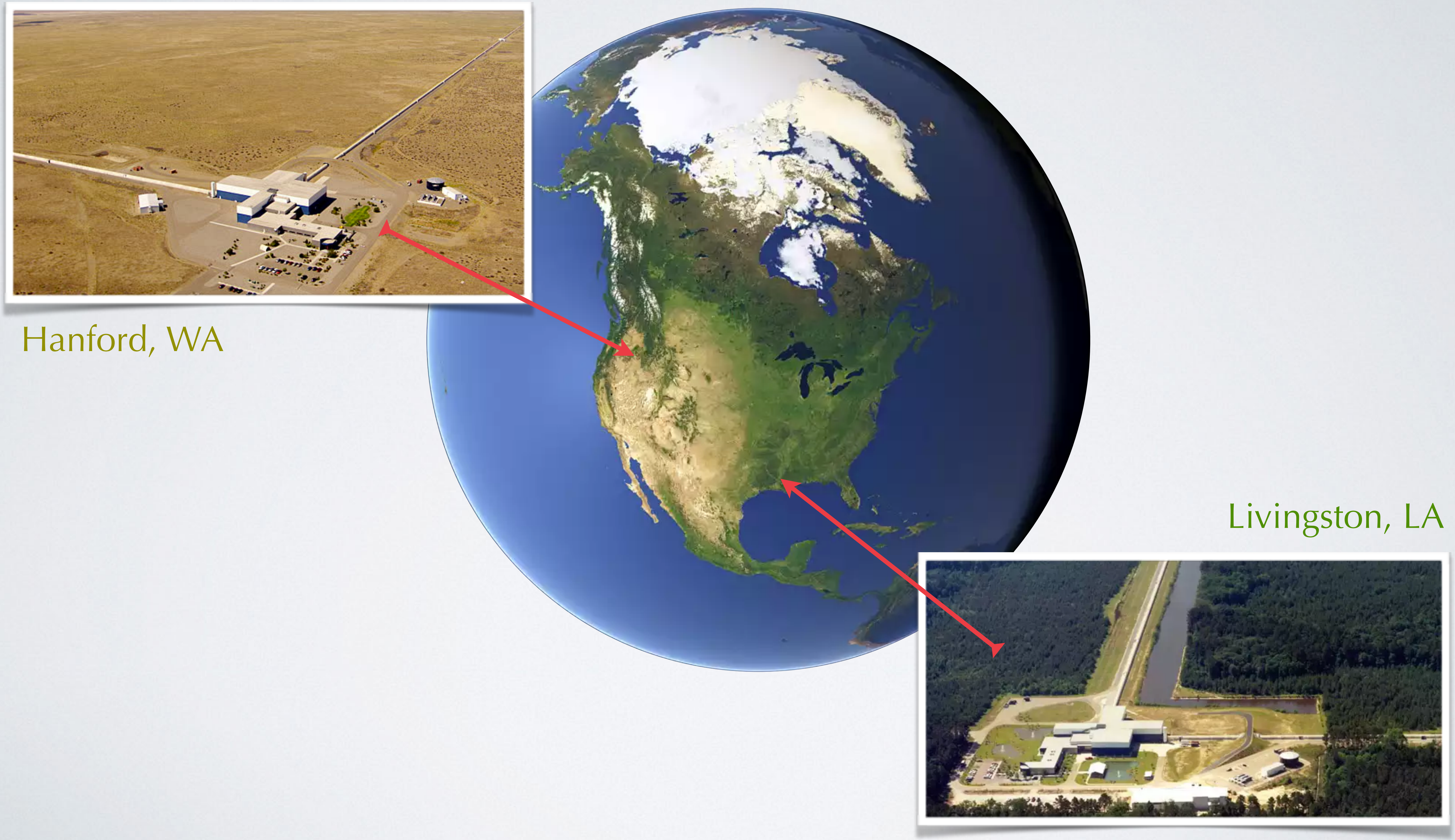

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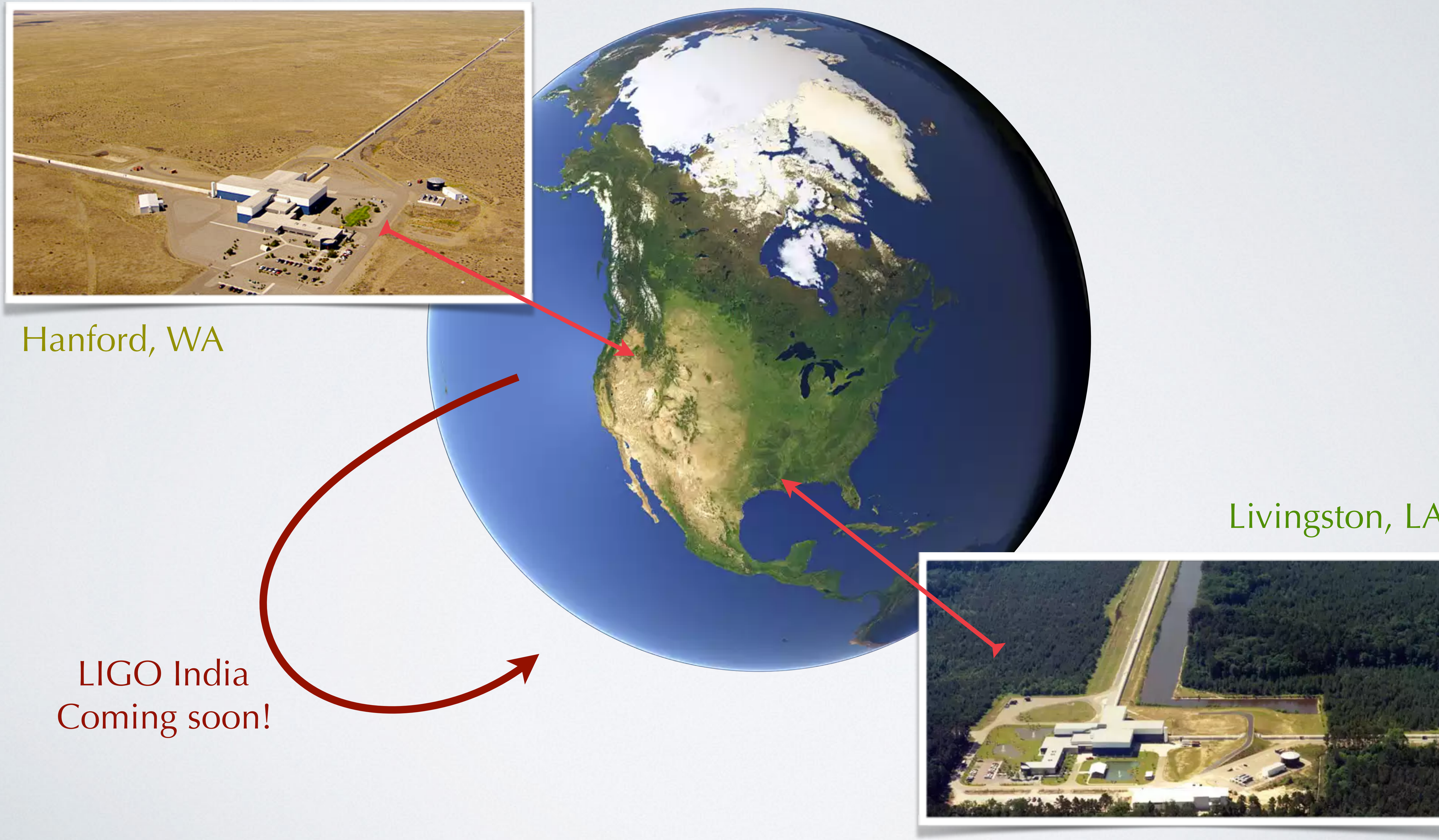
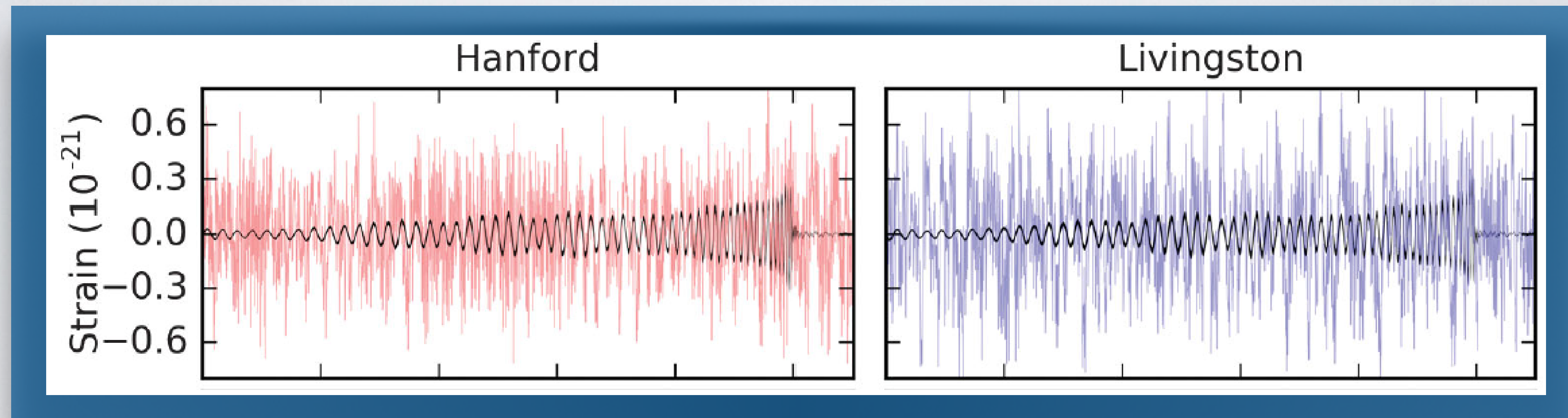
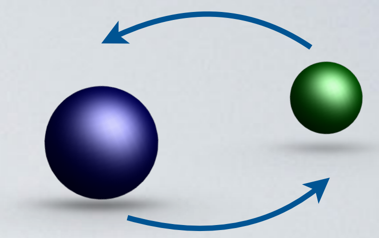


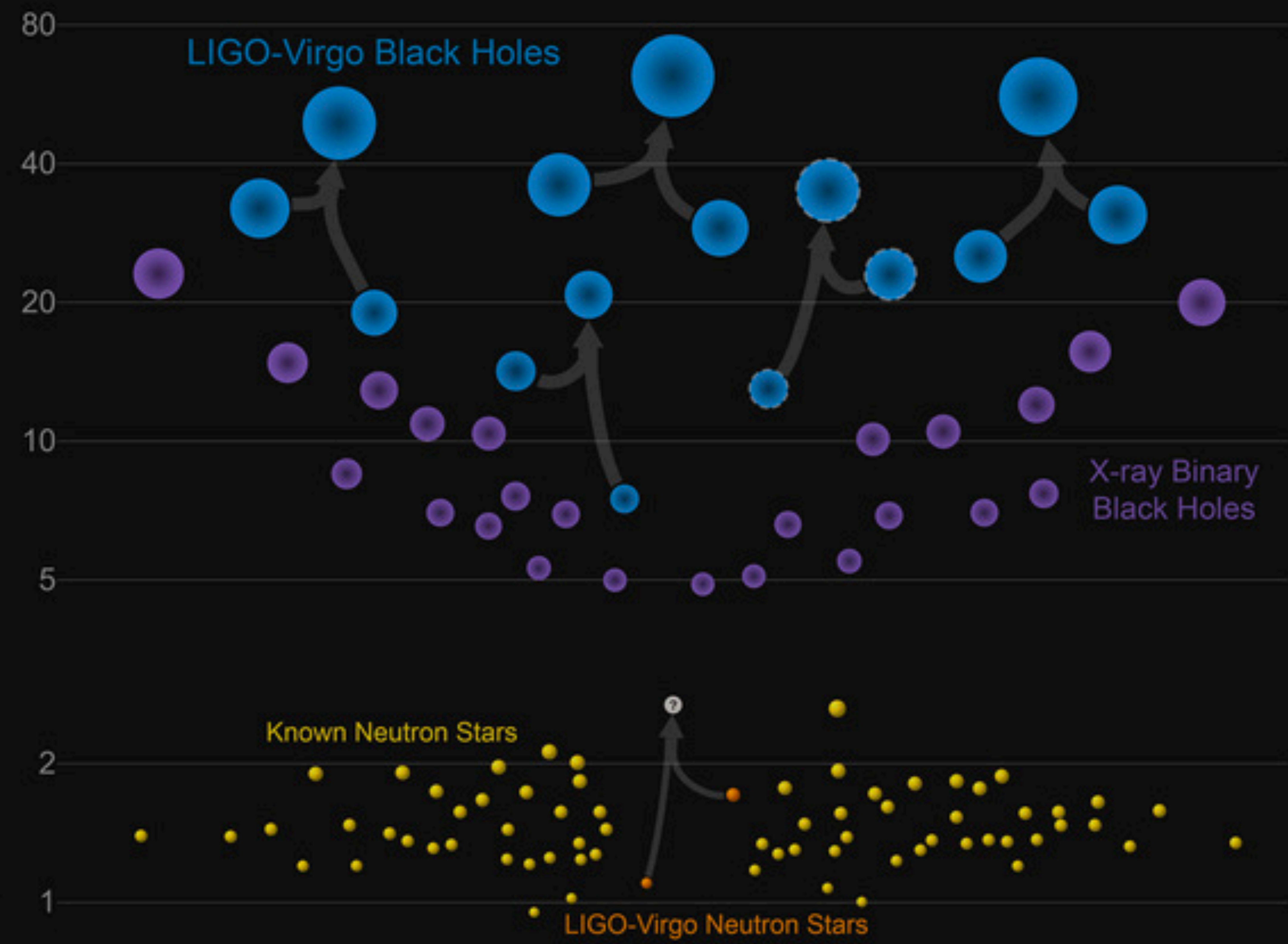
The signal is a needle in a haystack

Signal from the second detection, GW151226

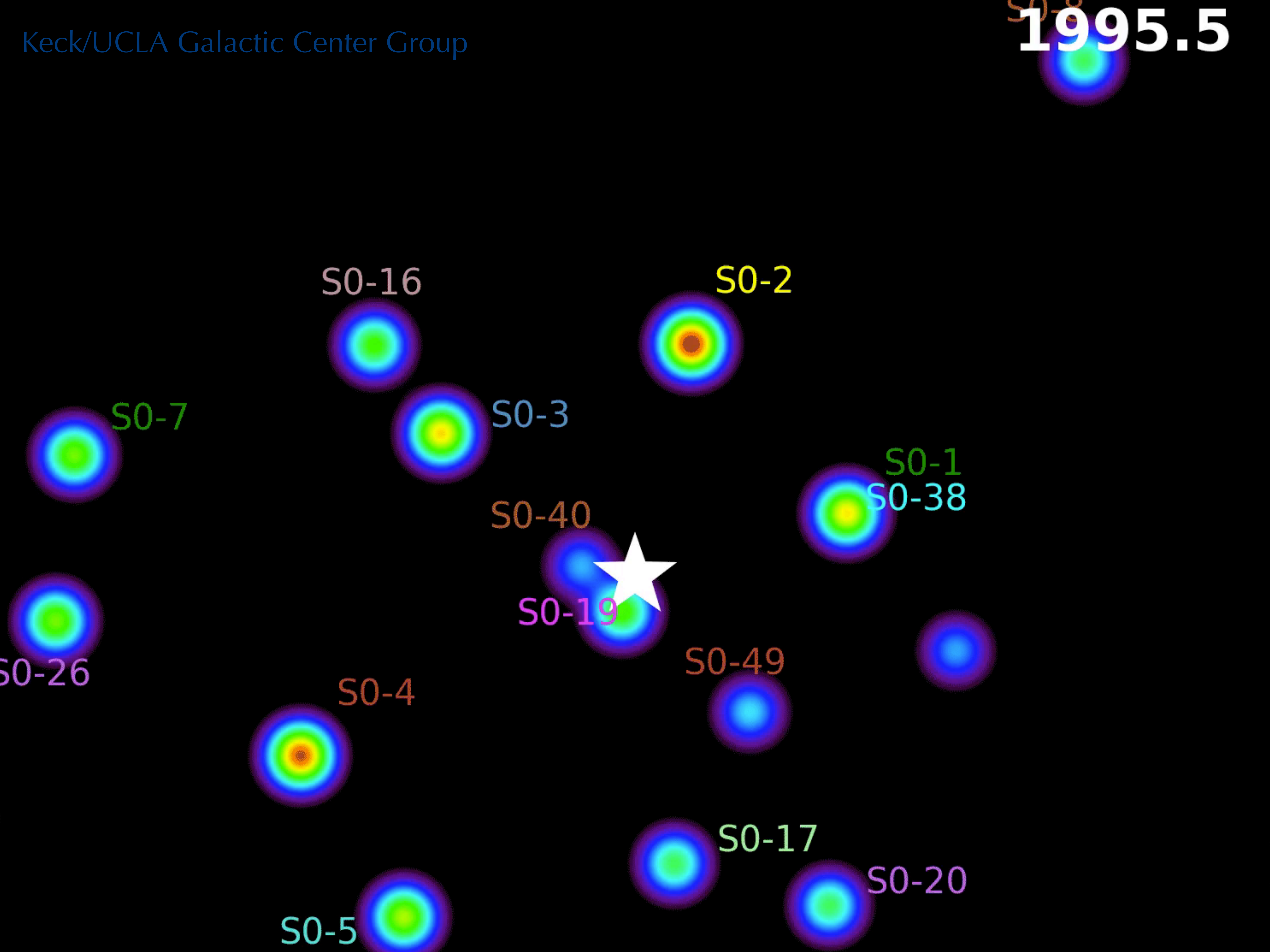


Known gravitational wave sources are myriad





Frank Elavsky


Keck/UCLA Galactic Center Group
1995.5
S0-16
S0-2
S0-7
S0-3
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S0-38
S0-40
S0-19
S0-49
S0-4
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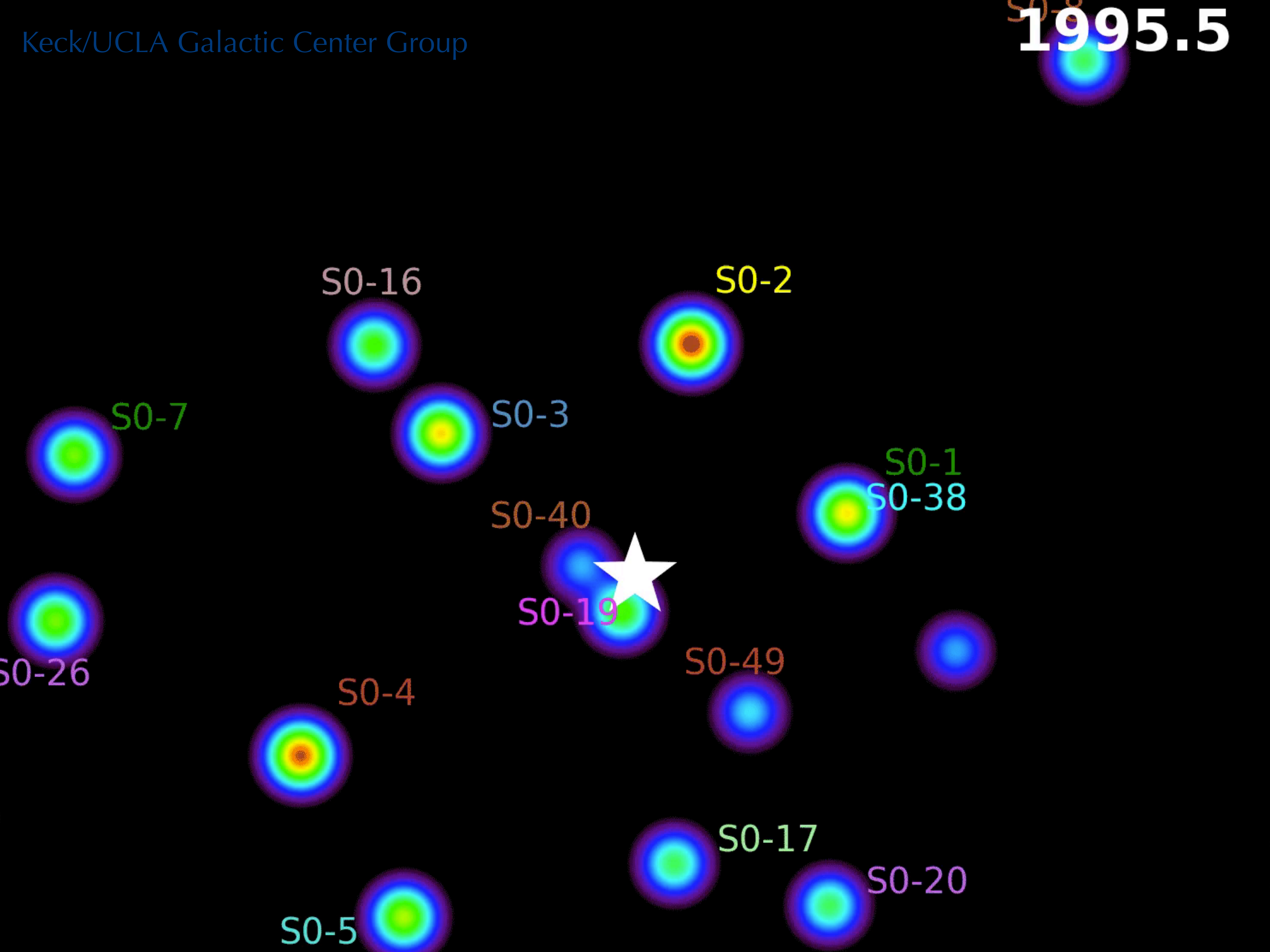
S0-20
S0-5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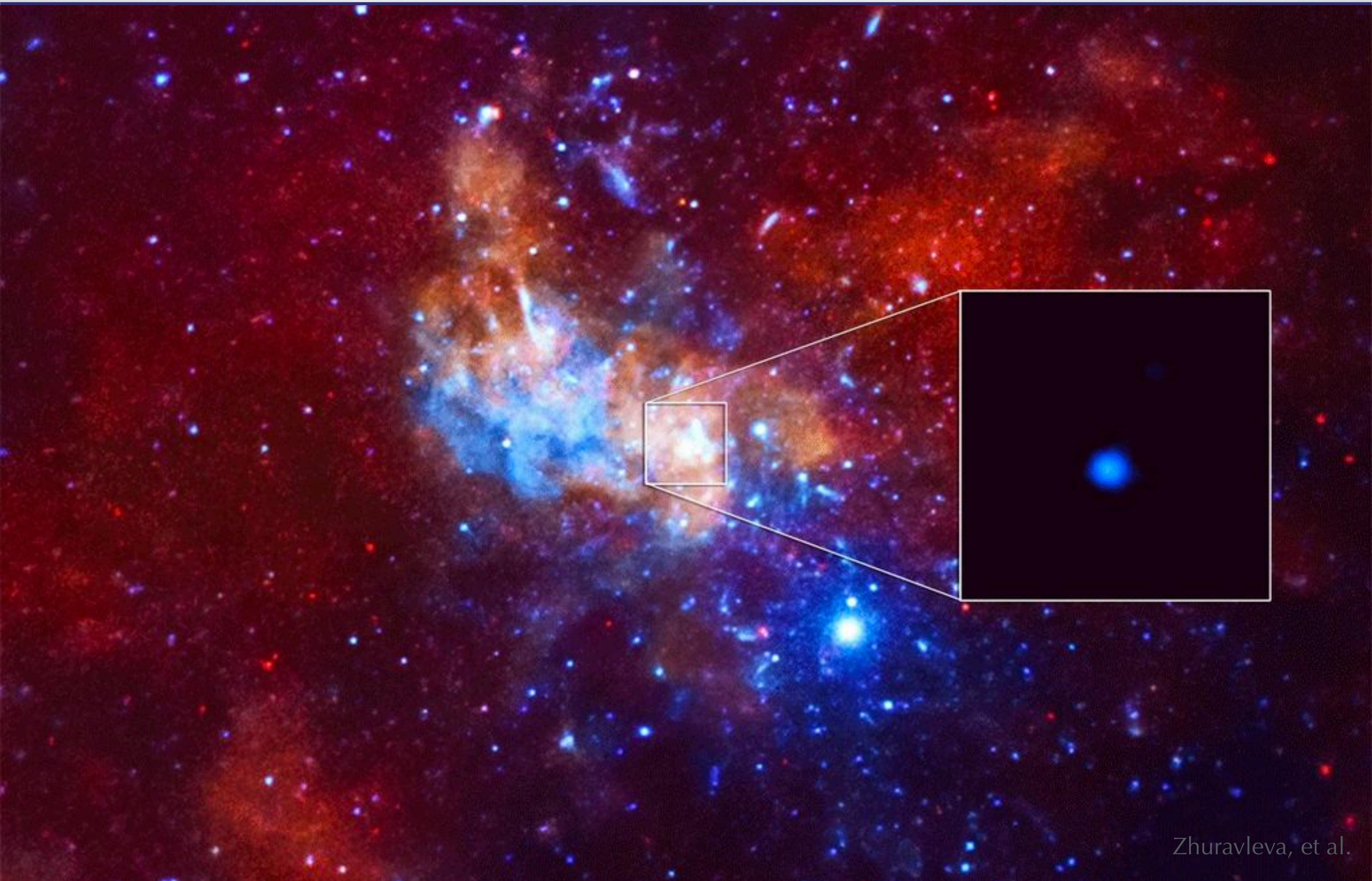
Sagittarius A* is about 4 million $M_{\odot}$



The black hole spectrum is large

Mass	Name

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Mass	Name
$2 \times 10^{11} M_\odot$	SDSS J140821.67+025733.2

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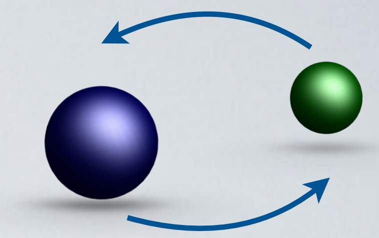
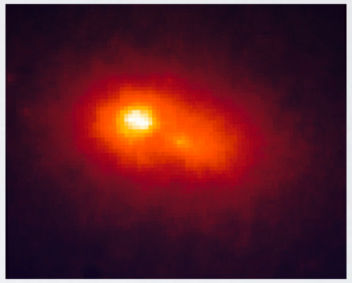


Mass	Name
$2 \times 10^{11}\ M_\odot$	SDSS J140821.67+025733.2
$1.5 \times 10^{8}\ M_\odot$	P2 (in Andromeda)



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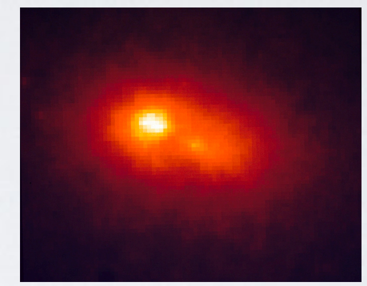

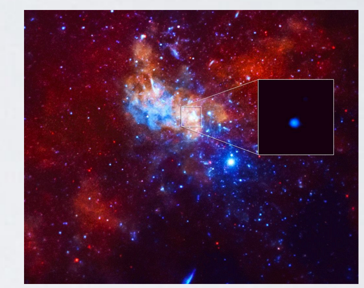

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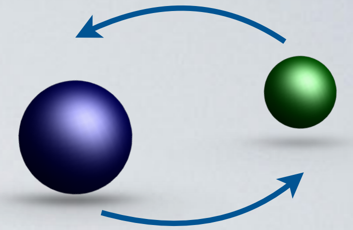

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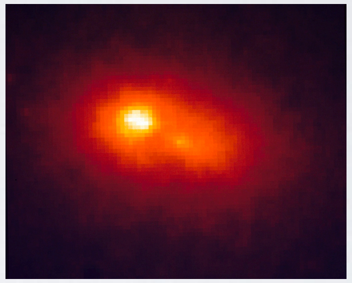
10^{2} - $10^{4}\ M_\odot$	IMBHs??



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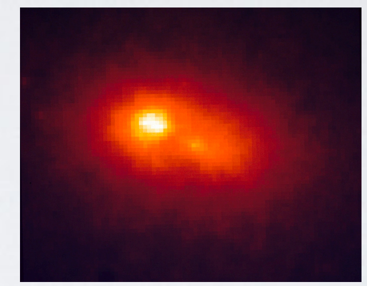
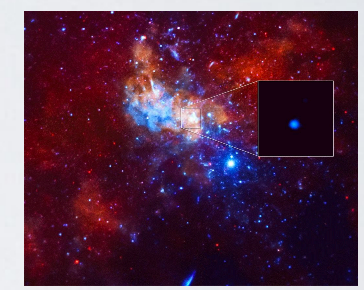
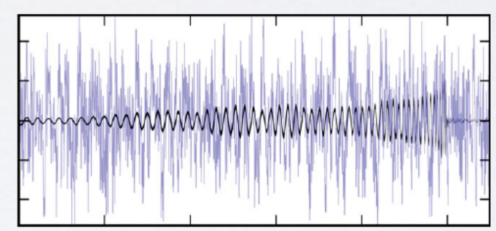
10 - 100 $M_\odot$	LIGO/Virgo BHs







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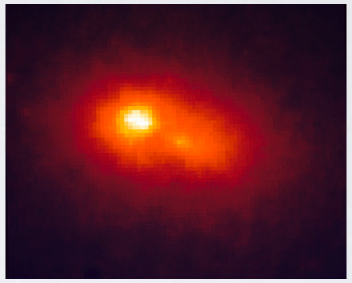

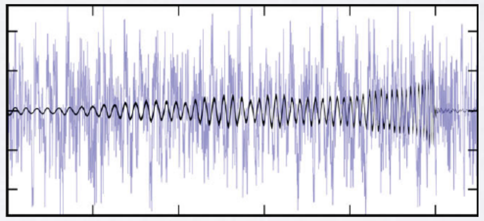

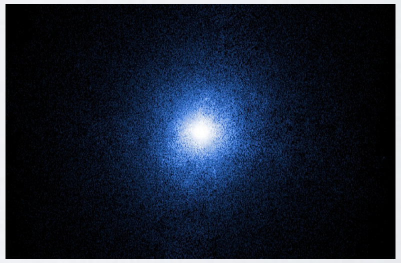

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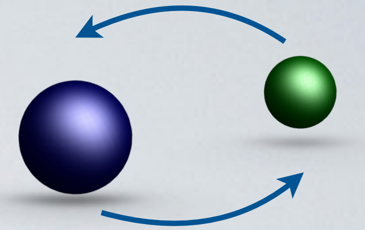

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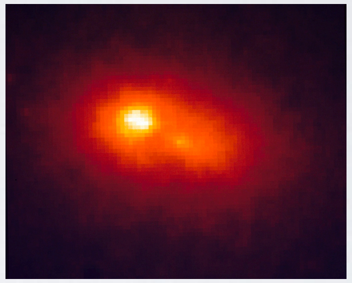
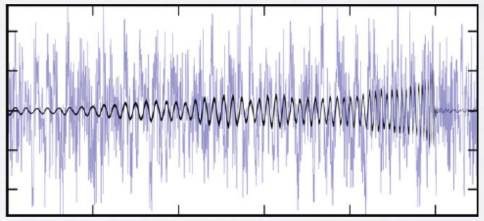
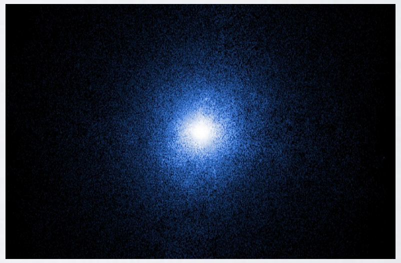
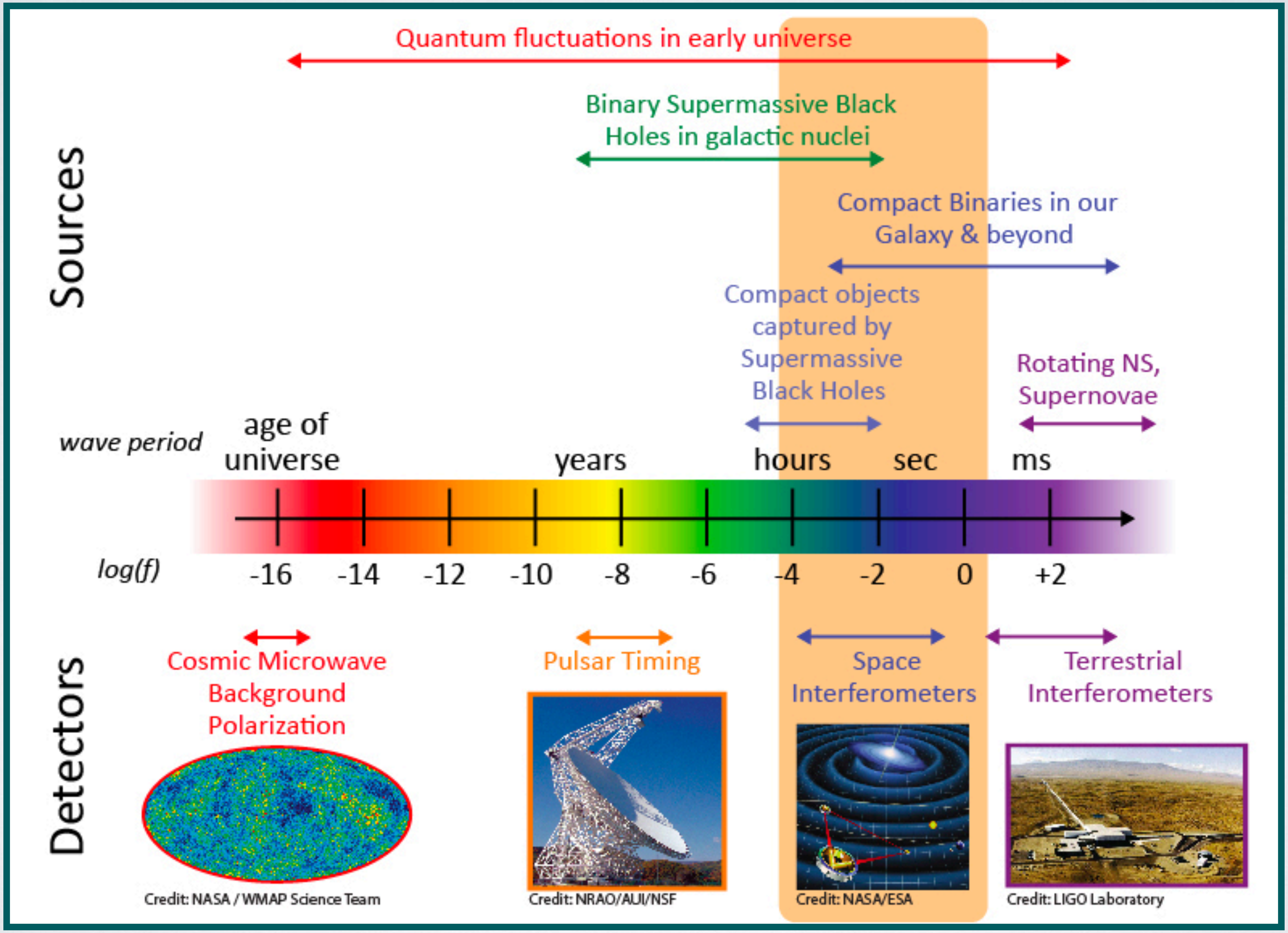
$10^{2} - 10^{4} M_\odot$	IMBHs??
$10 - 100 M_\odot$	LIGO/Virgo BHs
$5 - 20 M_\odot$	X-ray binaries
$10^{-??} M_\odot$	Micro black holes??





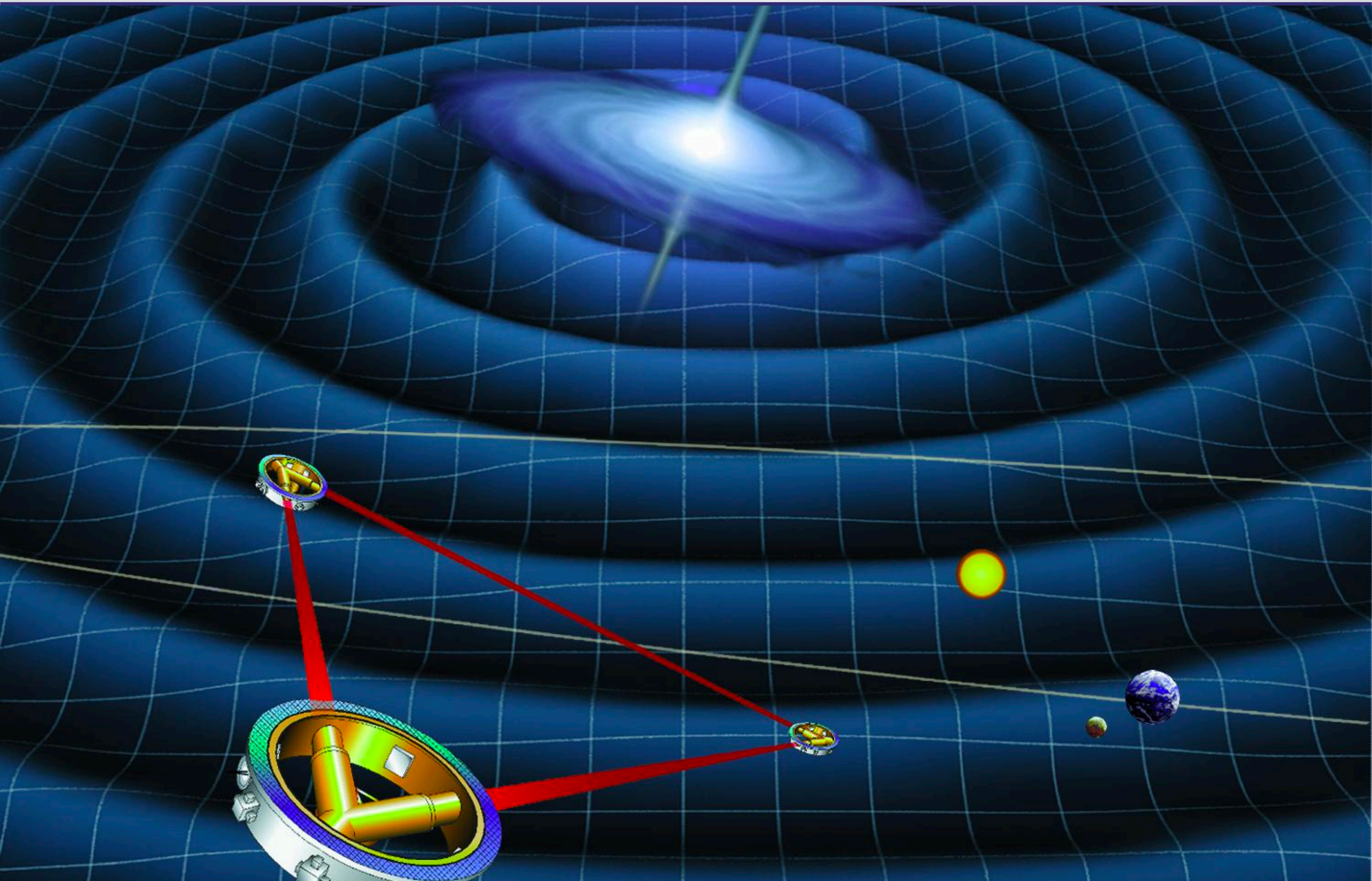


The gravitational wave spectrum is wide and full of wonders



lisa.nasa.gov

LISA is planned to launch in 2034 and directly observe EMRI (and IMRIs?)



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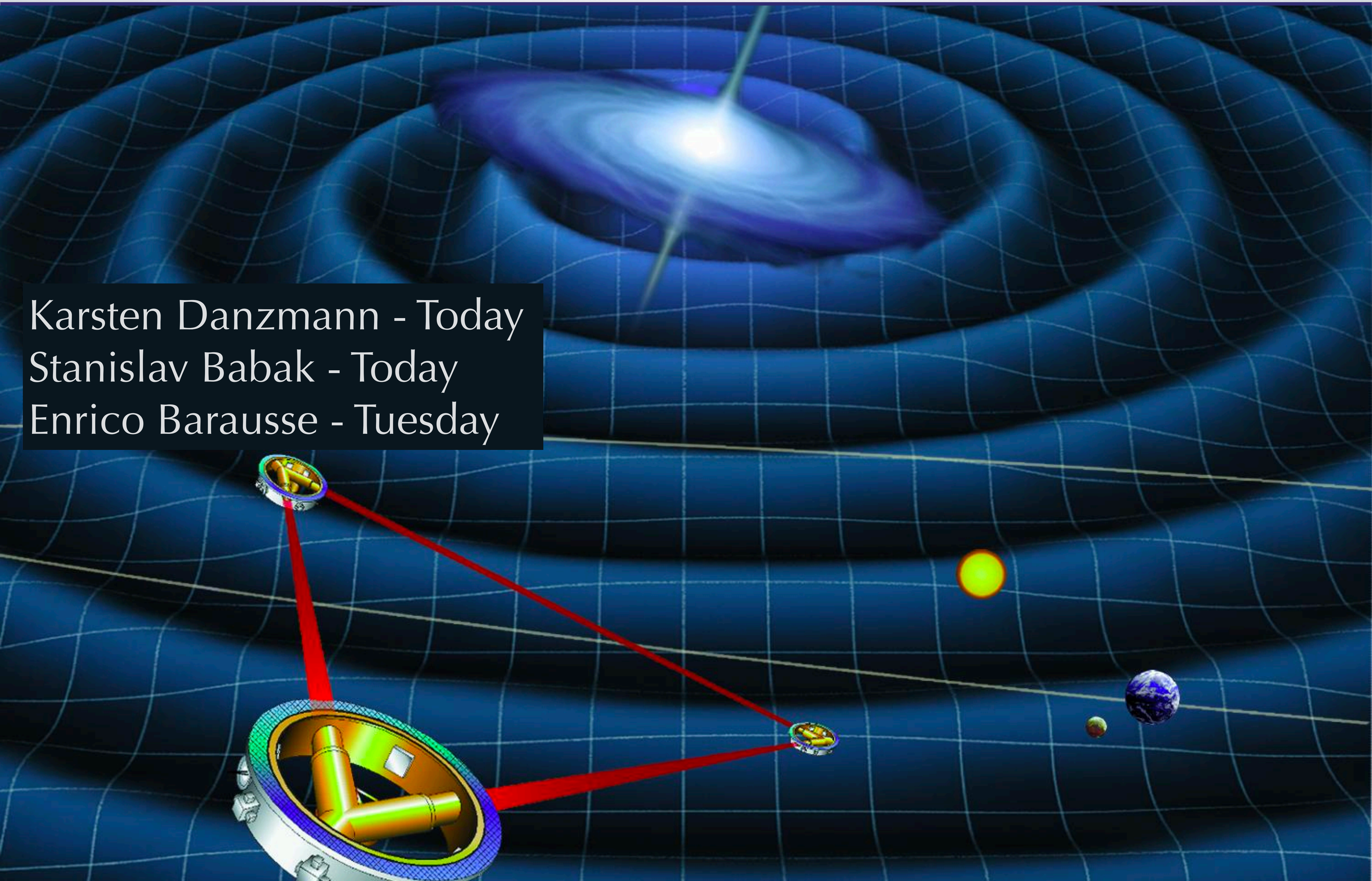

Karsten Danzmann - Today
Stanislav Babak - Today
Enrico Barausse - Tuesday

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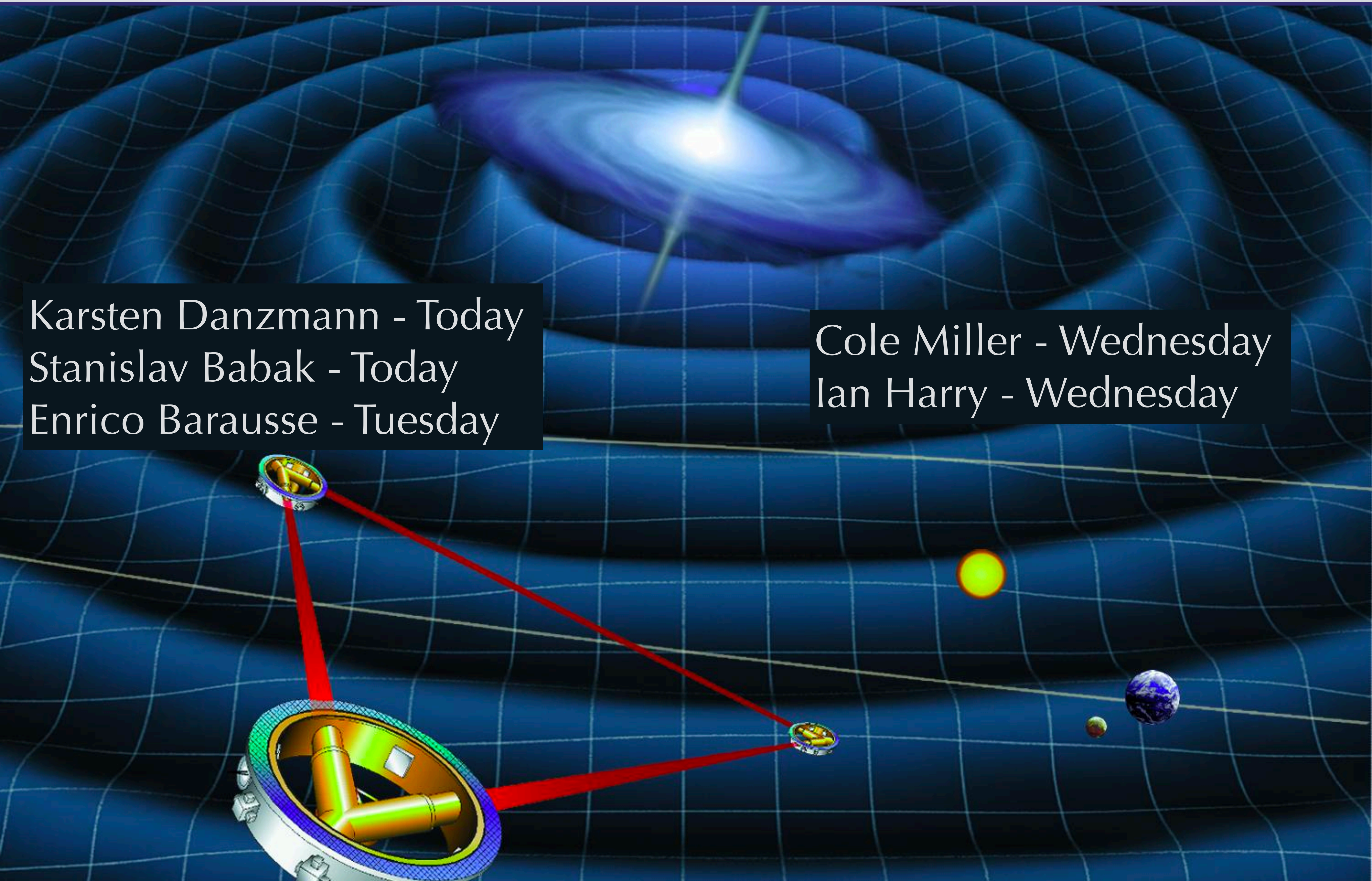

This is why we're here

$$\mathrm{g}_{\alpha\beta} = g^{\mathrm{BH}}_{\alpha\beta} + h^{1}_{\alpha\beta} + h^{2}_{\alpha\beta} + \cdots$$

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$$g_{\alpha\beta} = g^{\rm BH}_{\alpha\beta} + h^1_{\alpha\beta} + h^2_{\alpha\beta} + \cdots$$

Order	Field equations	Equations of motion
0	$G_{\mu\nu}[g_{\alpha\beta}] = 0$	
1		
2		

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Order	Field equations		Equations of motion
0	$G_{\mu\nu}[\mathrm{g}_{\alpha\beta}] = 0$	$\longrightarrow$	$\dfrac{D^2 z^\mu}{d\tau^2} = 0$
1			
2			

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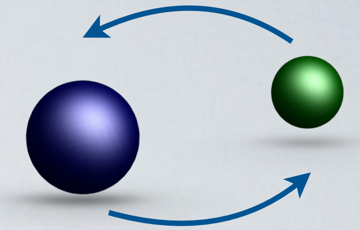

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Order	Field equations	Equations of motion
0	$G_{\mu\nu}[g_{\alpha\beta}] = 0$	$\frac{D^2 z^\mu}{d\tau^2} = 0$
1	$\Box \bar{h}^1_{\mu\nu} + 2R_{\alpha\mu\beta\nu}\bar{h}^{1\alpha\beta} = -16\pi T_{\mu\nu}$	
2		

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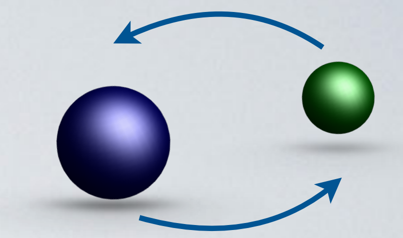

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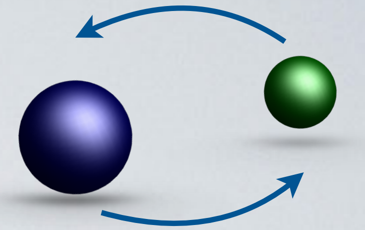

$$\mathrm{g}_{\alpha\beta} = g^{\mathrm{BH}}_{\alpha\beta} + h^1_{\alpha\beta} + h^2_{\alpha\beta} + \cdots$$

Order	Field equations	Equations of motion
0	$G_{\mu\nu}[\mathrm{g}_{\alpha\beta}] = 0$	$\dfrac{D^2 z^\mu}{d\tau^2} = 0$
1	$\Box \bar{h}^1_{\mu\nu} + 2R_{\alpha\mu\beta\nu}\bar{h}^{1\alpha\beta} = -16\pi T_{\mu\nu}$	$\dfrac{D^2 z^\mu}{d\tau^2} = F^\mu_{\mathrm{self}}[h^1_{\alpha\beta}]$
2	$\Box h^2 = (1 + h^1)T[z + \delta z] + (\nabla h^1)^2$	

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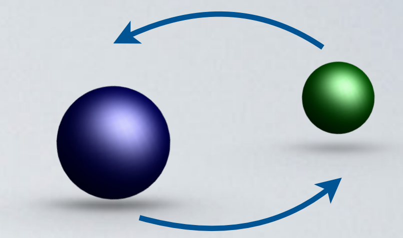

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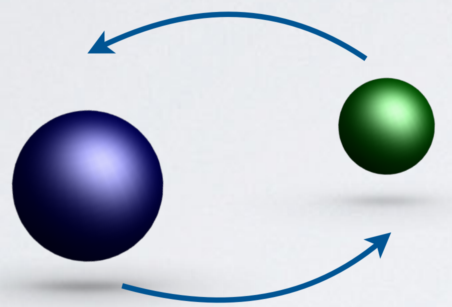
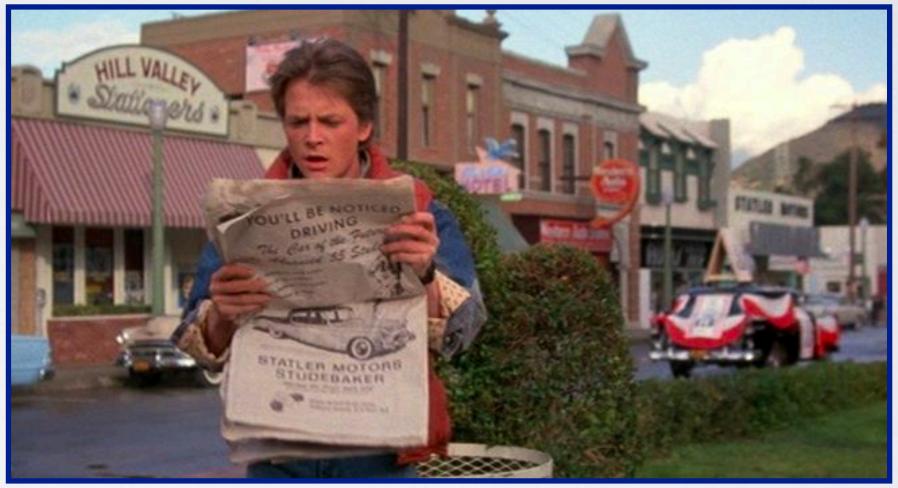
2	$\Box h^2 = (1 + h^1)T[z + \delta z] + (\nabla h^1)^2$	

Outline

Why we're here



Yesterday



Today

Tomorrow

Outline

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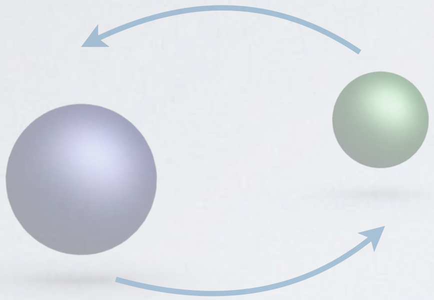
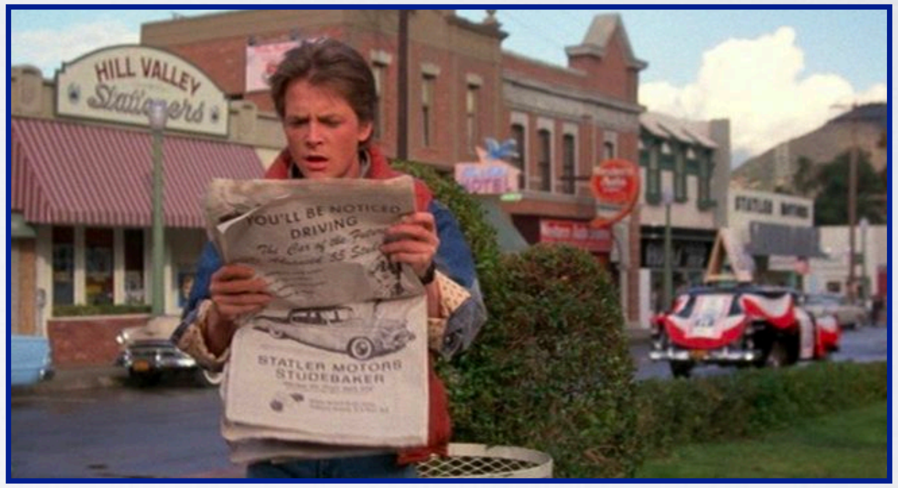


Yesterday



Today

Tomorrow

Yesterday

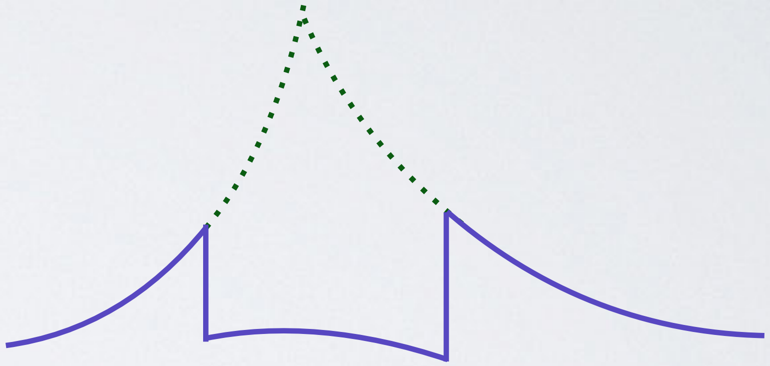
Yesterday

History

Yesterday

History

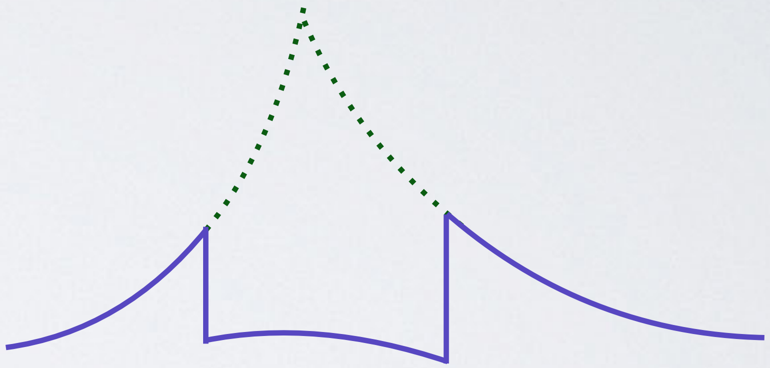
Regularization



Yesterday

History

Regularization

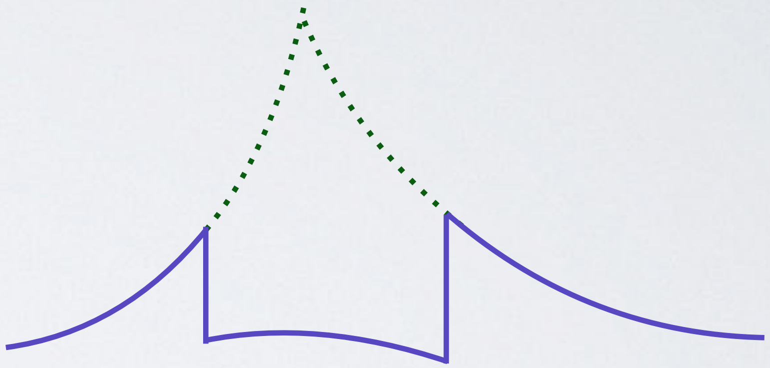


Practical considerations

Yesterday

History

Regularization



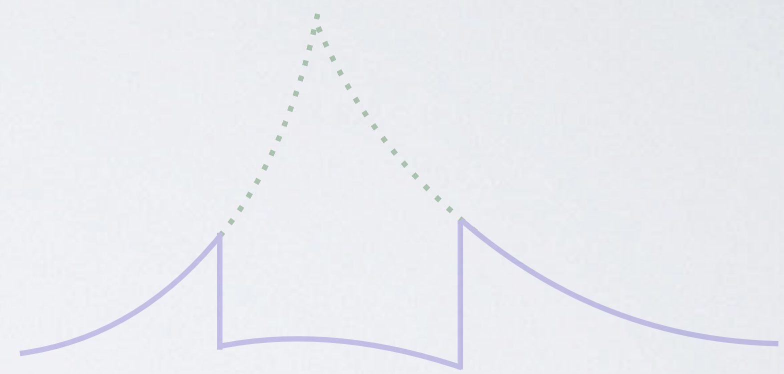
Practical considerations

Gauge invariants

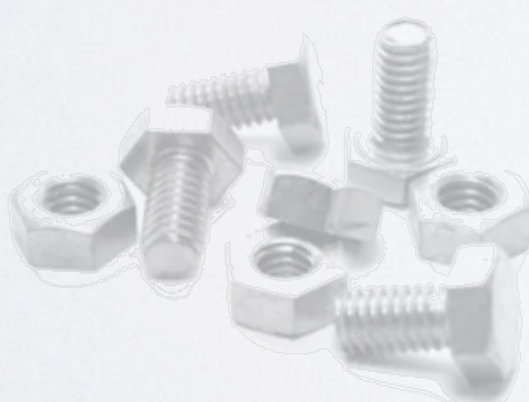
Yesterday

History

Regularization

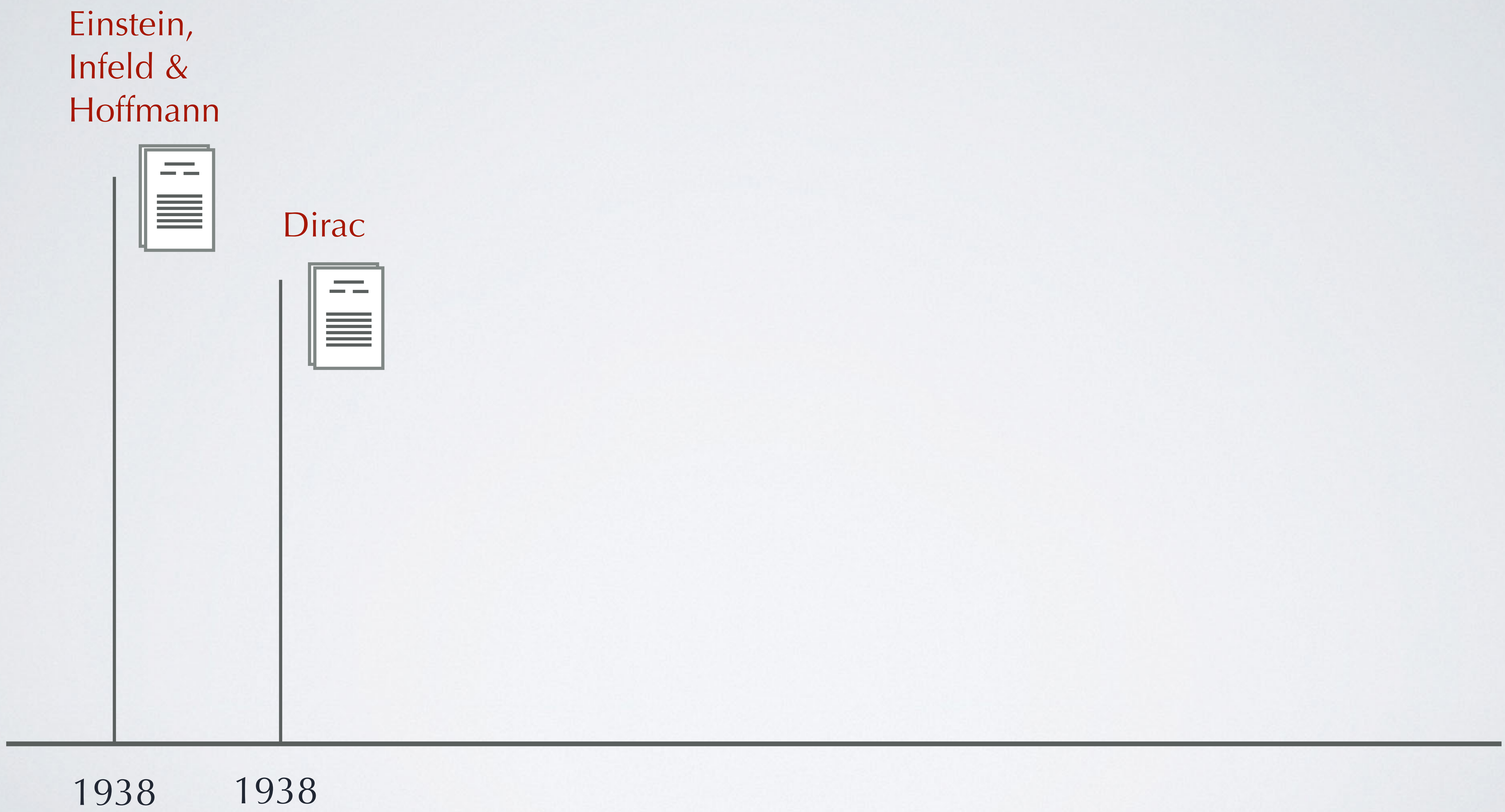


Practical considerations



Gauge invariants

Self-force research dates back to the 1930s



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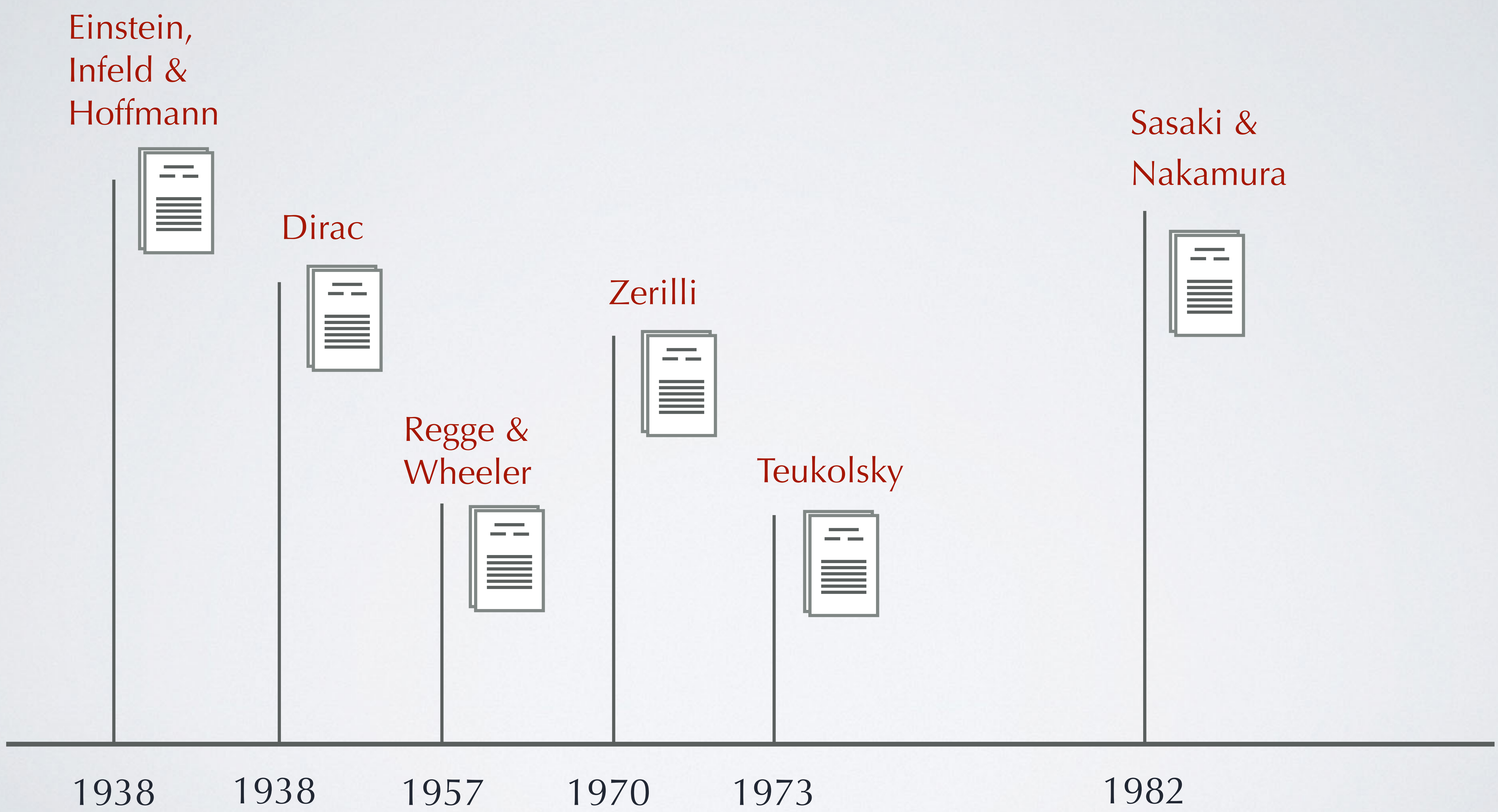

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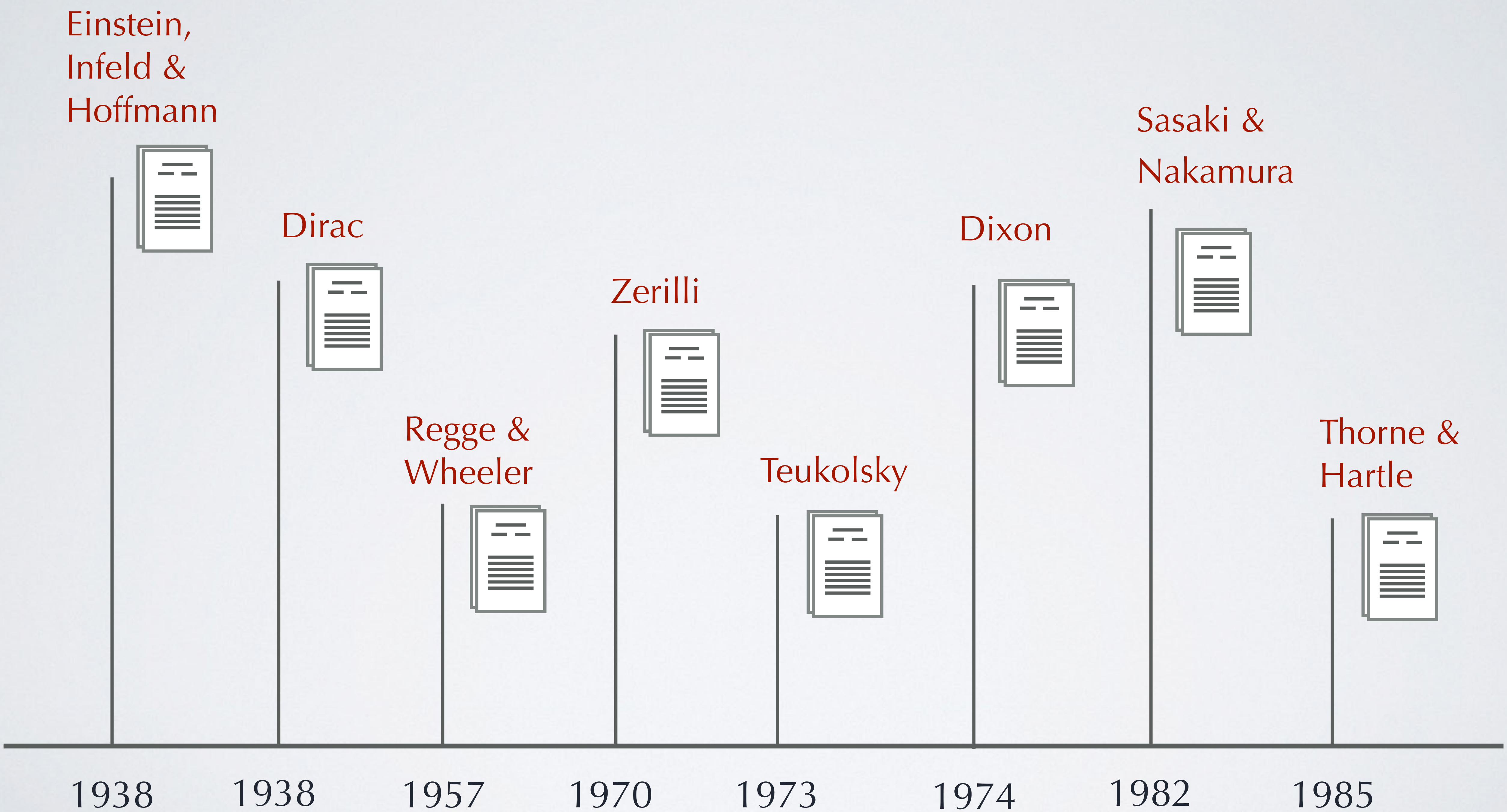

A self-force breakthrough was made in 1996 by Mino, Sasaki & Tanaka, and also Quinn & Wald

Gravitational Radiation Reaction to a Particle Motion

Yasushi Mino,[1,2*] Misao Sasaki,[1†] and Takahiro Tanaka[1‡]

[1] *Department of Earth and Sp*
Osaka Univers
[2] *Department of Physics, Faculty of S*

A small mass particle traveling in a curv
in the lowest order approximation with resp

An axiomatic approach to electromagnetic and gravitational radiation reaction of particles in curved spacetime

Theodore C. Quinn and Robert M. Wald

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Detweiler and Whiting found a particularly convenient split of the retarded field

Self force via a Green's function decomposition

Steven Detweiler and Bernard F. Whiting
Department of Physics, PO Box 118440, University of Florida, Gainesville, FL 32611-8440
(Dated: November 12, 2002)

The gravitational field in a neighborhood of a particle of small mass μ moving through curved spacetime is naturally decomposed into two parts each of which satisfies the perturbed Einstein equations through $O(\mu)$. One part is an inhomogeneous field which looks like the μ/r field tidally distorted by the local Riemann tensor. The other part is a homogeneous field that completely determines the self force of the particle interacting with its own gravitational field, which changes the worldline at $O(\mu)$ and includes the effects of radiation reaction. Surprisingly, a local observer measuring the gravitational field in a neighborhood of a freely moving particle sees geodesic motion of the particle in a perturbed vacuum geometry and would be unaware of the existence of radiation at $O(\mu)$. In the light of all previous work this is quite an unexpected result.

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$$\Box\, h^{\text{ret}} = 4\pi\rho \qquad\qquad h^{\text{ret}} = h^S + h^R$$

Singular	Regular
$\Box\, h^S = 4\pi\rho$	$\Box\, h^R = 0$
$\partial_\alpha h^S = 0$	$F_\alpha = \partial_\alpha h^R$
h^S can be found analytically through Herculean efforts	Particle moves on geodesic of spacetime $g + h^R$

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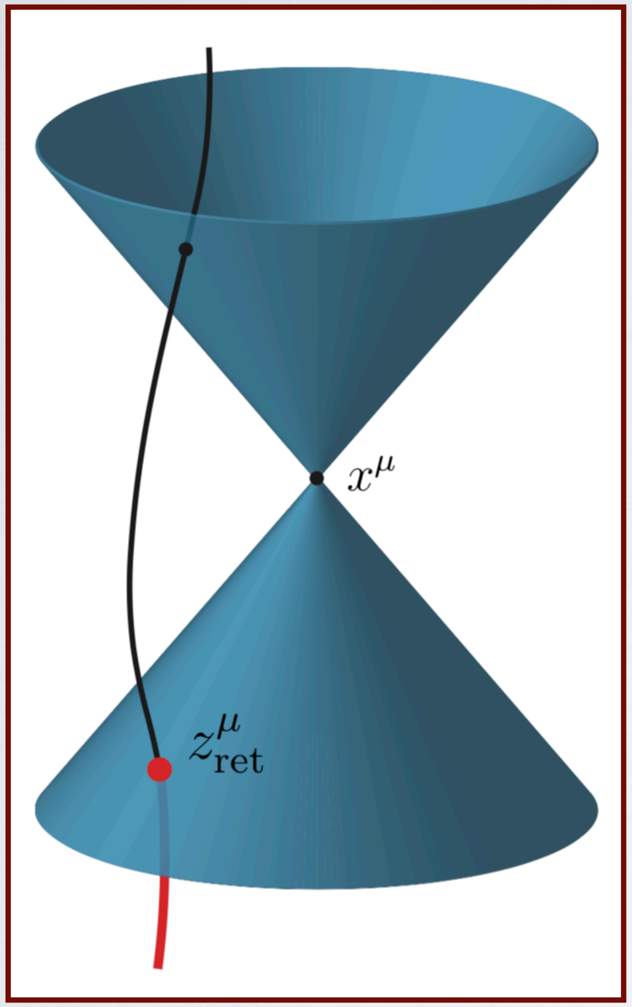
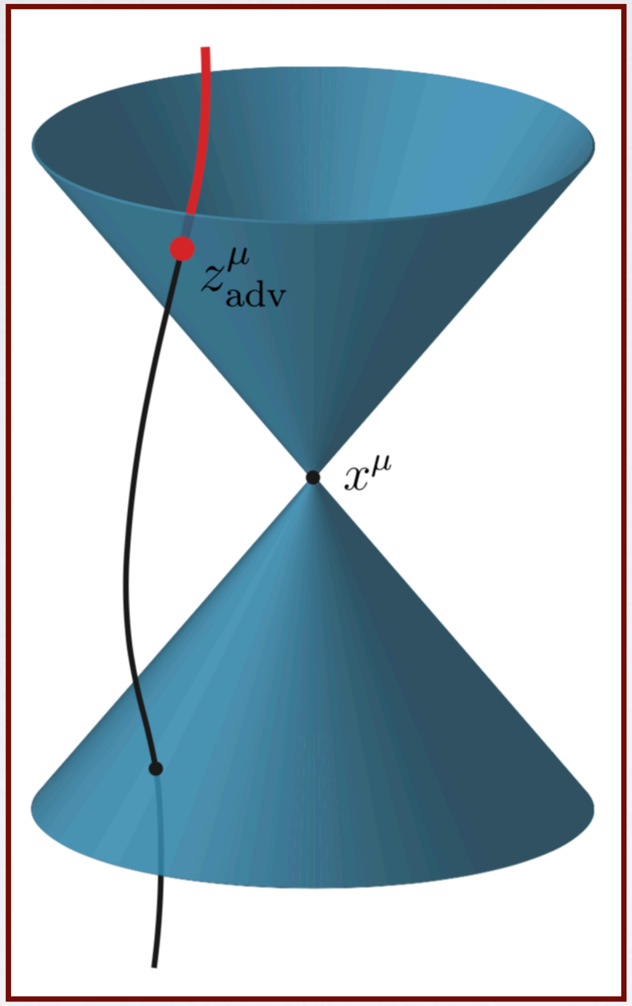


Retarded

$$G^{\mu}_{\text{ret}\ \mu'}$$



Advanced

$$G^{\mu}_{\text{adv}\ \mu'}$$

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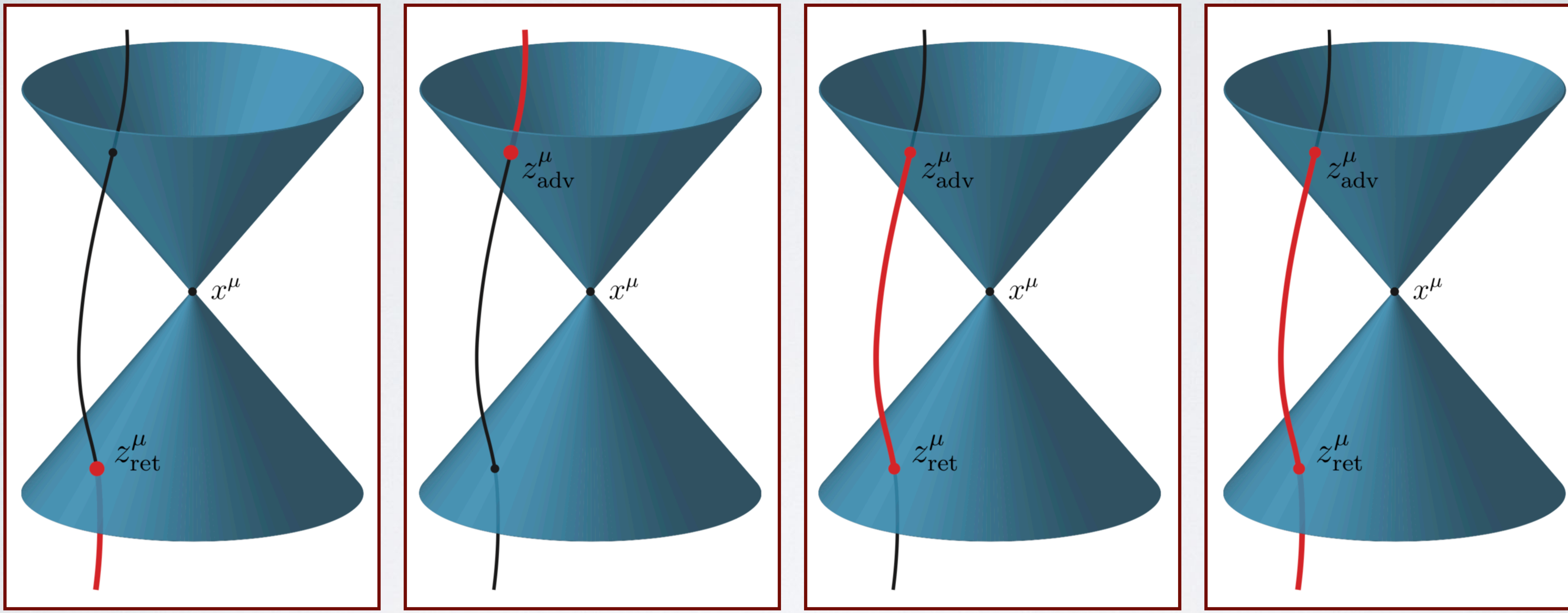


Retarded — $G^\mu_{\text{ret }\mu'}$

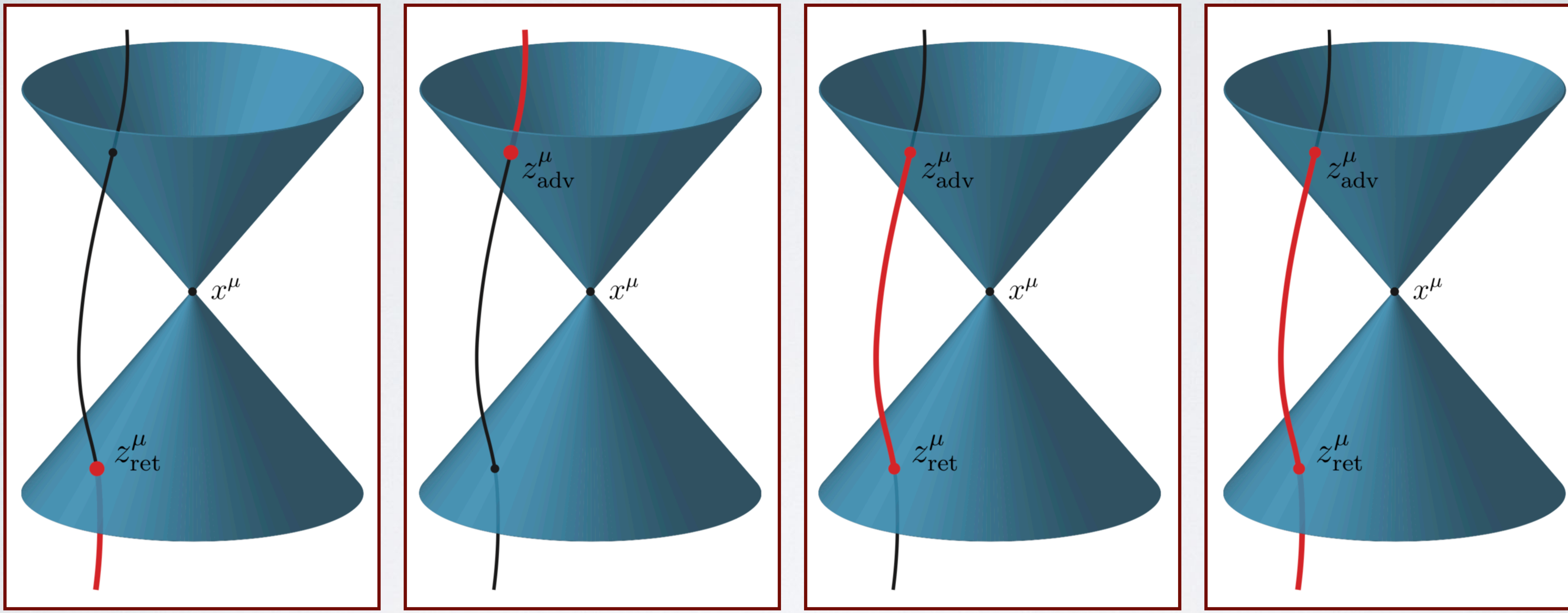
Advanced — $G^\mu_{\text{adv }\mu'}$

Singular — $G^\mu_{S\,\mu'}$

Regular — $G^\mu_{R\,\mu'}$

Detweiler and Whiting found a particularly convenient split of the retarded field

Barack & Pound, 2018



Retarded	Advanced	Singular	Regular
$G^{\mu}_{\ \rm ret\ \mu'}$	$G^{\mu}_{\ \rm adv\ \mu'}$	$G^{\mu}_{\ S\,\mu'}$	$G^{\mu}_{\ R\,\mu'}$

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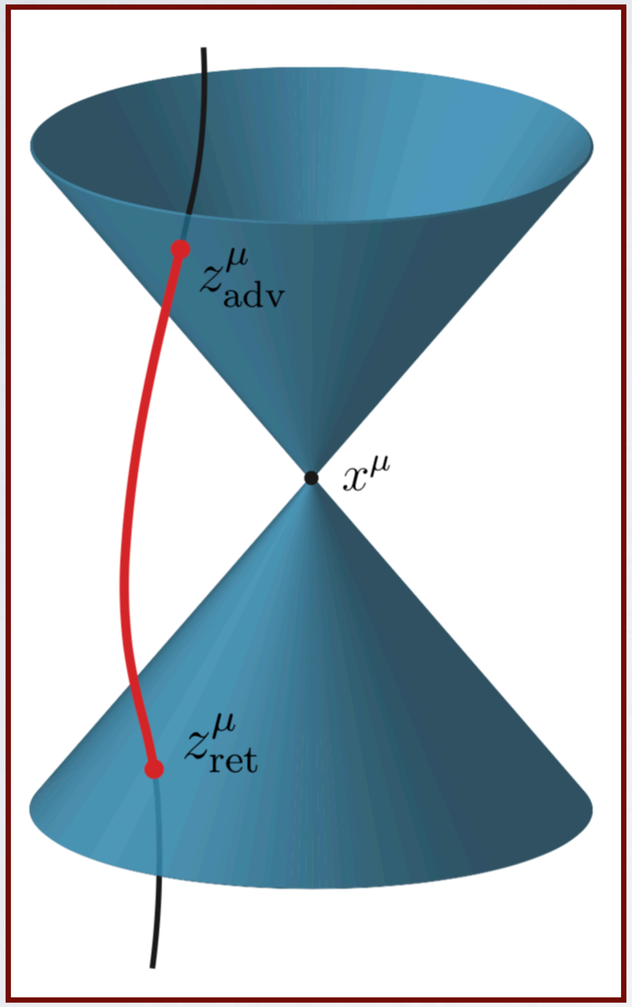
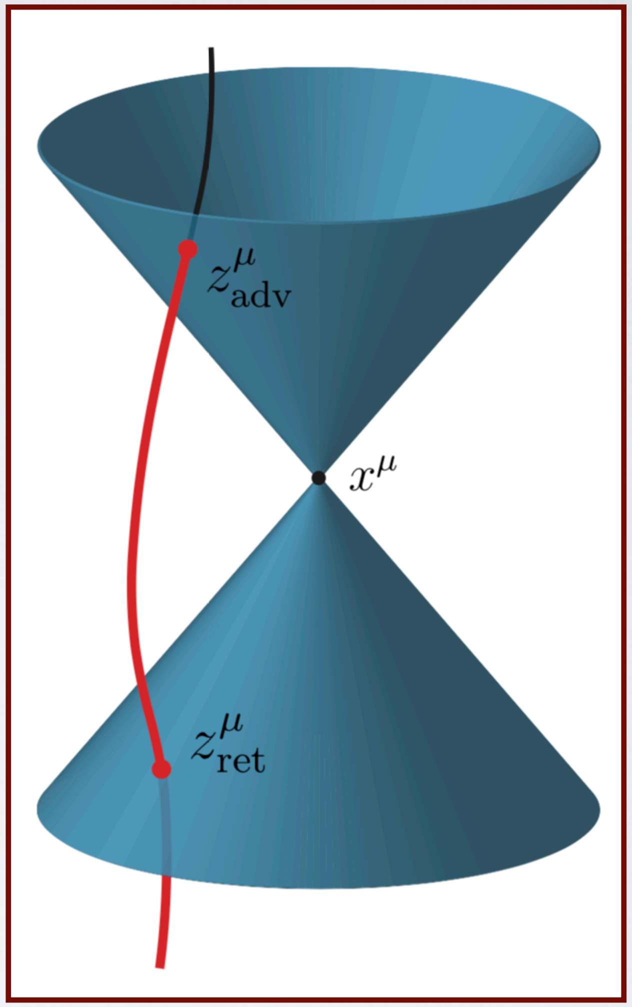


Singular



Regular

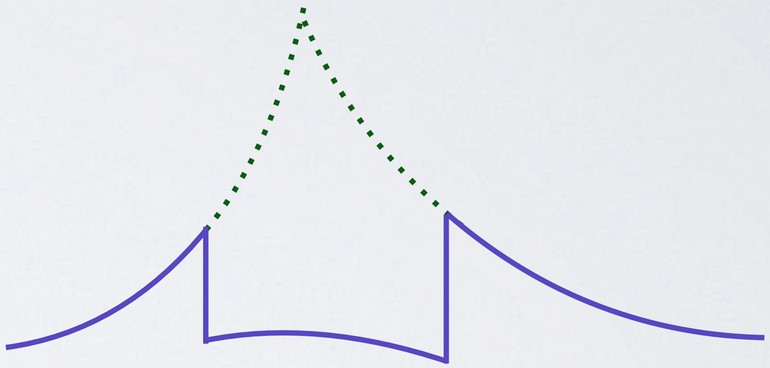
$$G^{\mu}_{S\,\mu'} = \frac{1}{2}\left(G^{\mu}_{\text{ret}\ \mu'} + G^{\mu}_{\text{adv}\ \mu'} - H^{\mu}{}_{\mu'}\right)$$

$$G^{\mu}_{R\,\mu'} = \frac{1}{2}\left(G^{\mu}_{\text{ret}\ \mu'} - G^{\mu}_{\text{adv}\ \mu'} + H^{\mu}{}_{\mu'}\right)$$

Yesterday

History

Regularization



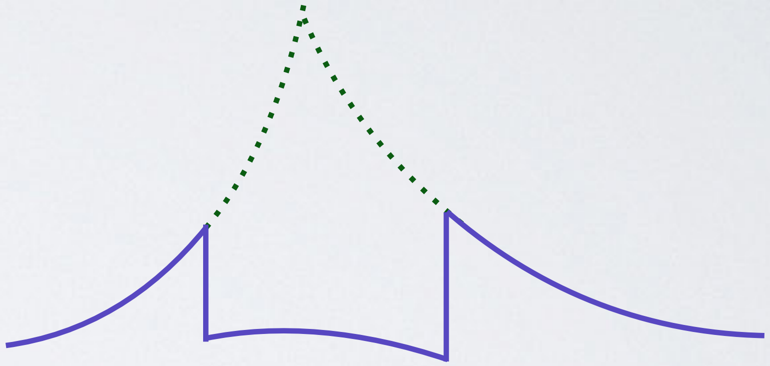
Practical considerations

Gauge invariants

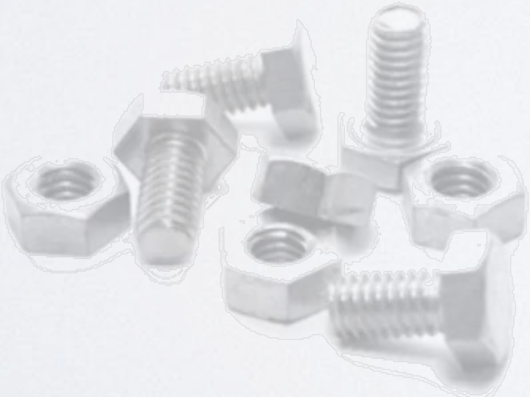
Yesterday

History

Regularization



Practical considerations



Gauge invariants

This is how to irritate an undergrad

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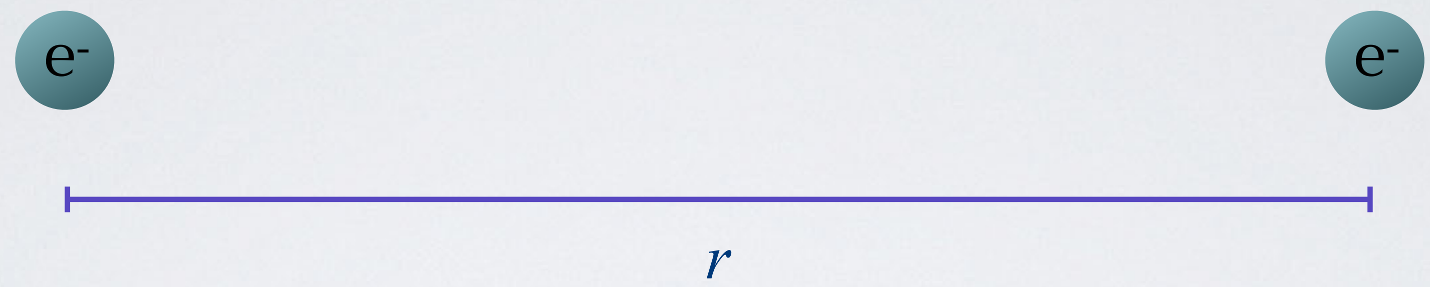

$$F = \frac{kq^2}{r^2}$$

This is how to irritate an undergrad

$$F = \frac{kq^2}{r^2} \qquad F = Eq$$

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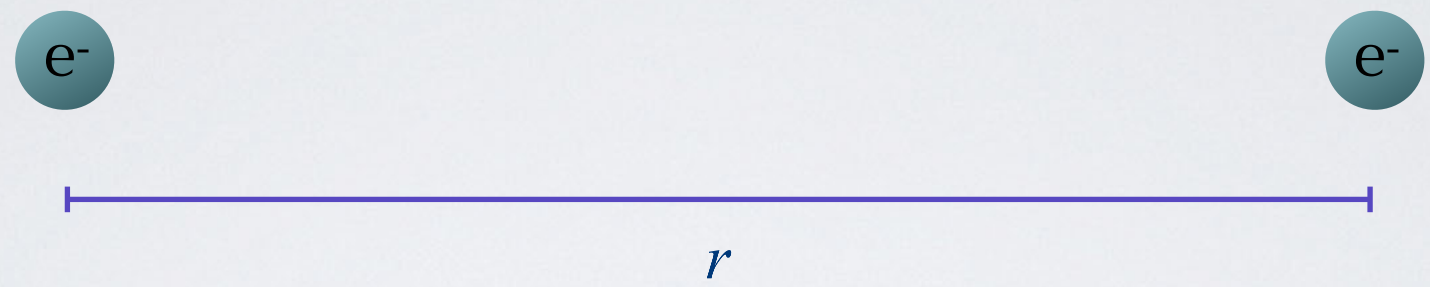

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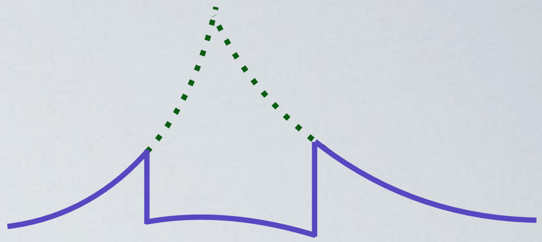

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3 Outcomes

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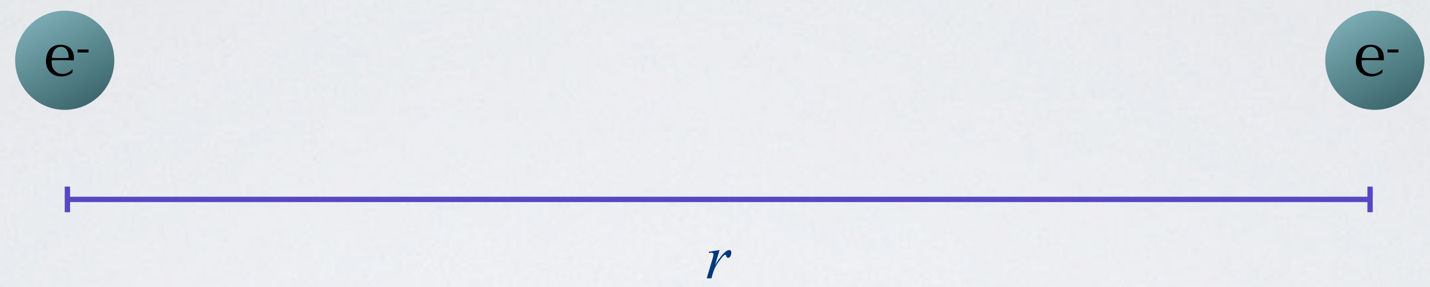
3 Outcomes

1. Becomes a chemistry major

This is how to irritate an undergrad

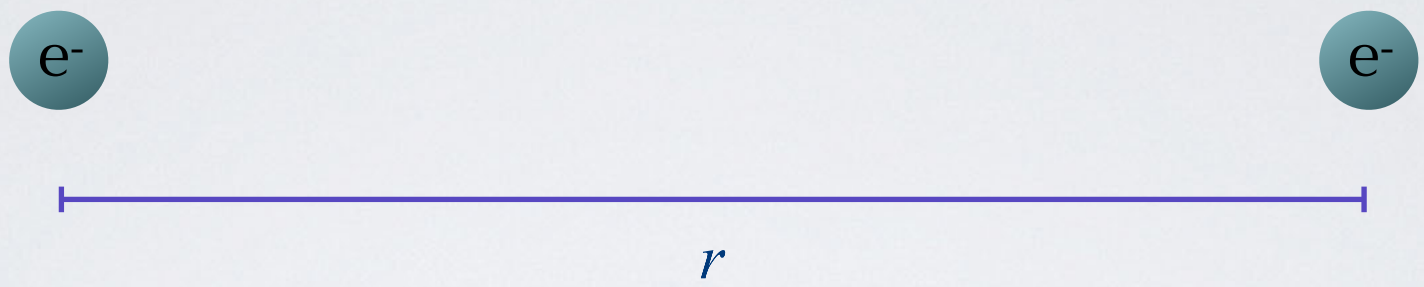


$$F = \frac{kq^2}{r^2} \qquad F = Eq \qquad E = \frac{kq}{r^2}$$

3 Outcomes

1. Becomes a chemistry major
2. Becomes a physics major

This is how to irritate an undergrad



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3 Outcomes

1. Becomes a chemistry major
2. Becomes a physics major
3. Becomes a philosophy major

There are three methods for performing regularization

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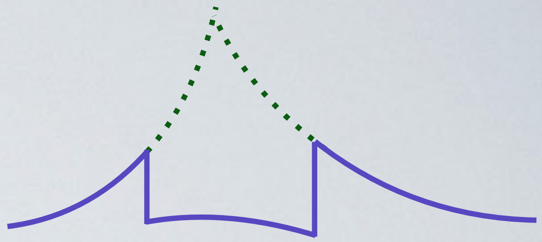
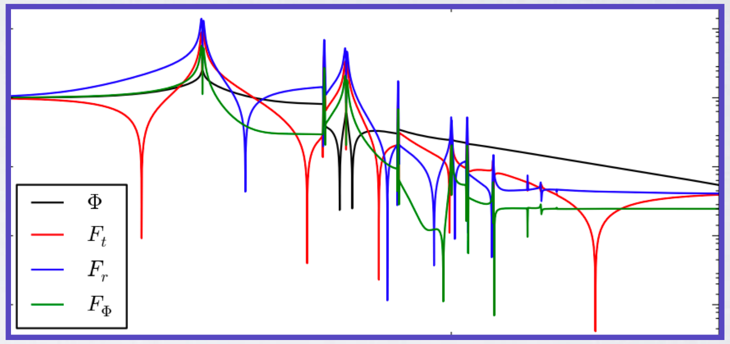
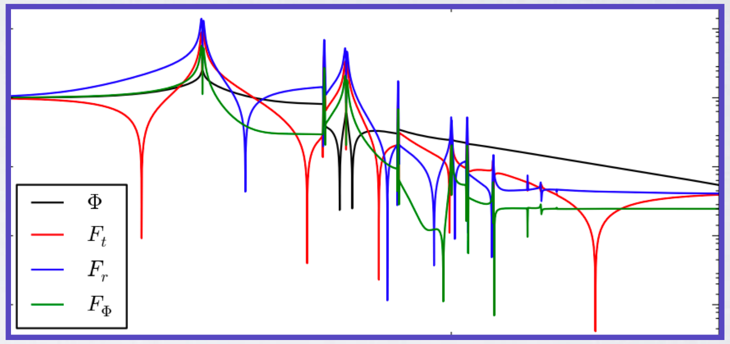




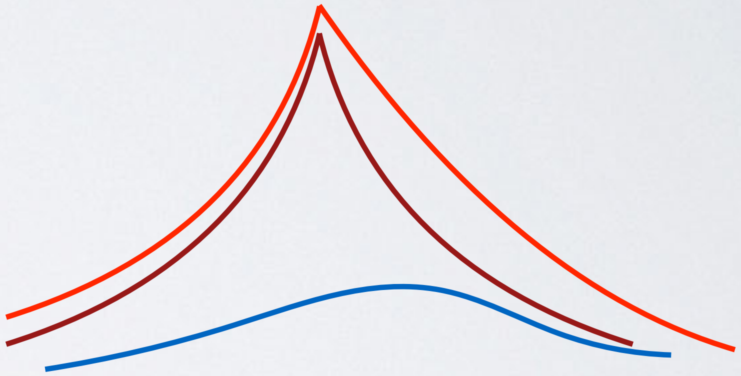
1. Worldline convolution

There are three methods for performing regularization



1. Worldline convolution

2. Mode-sum regularization



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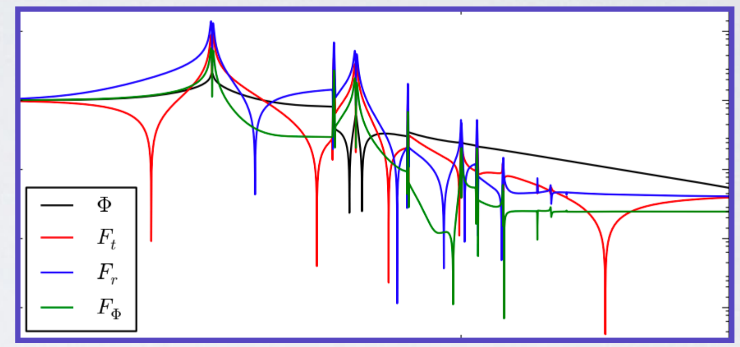

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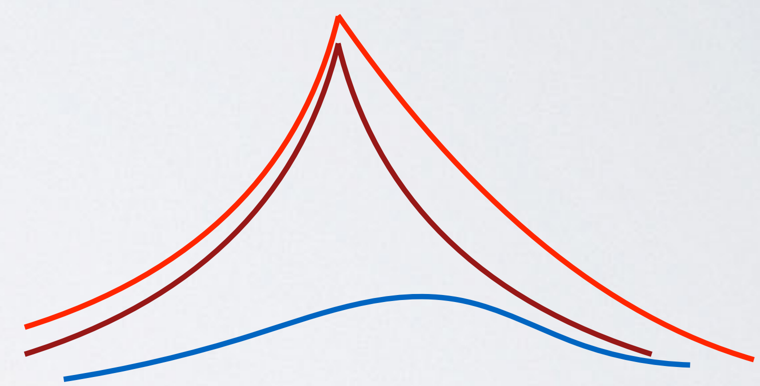
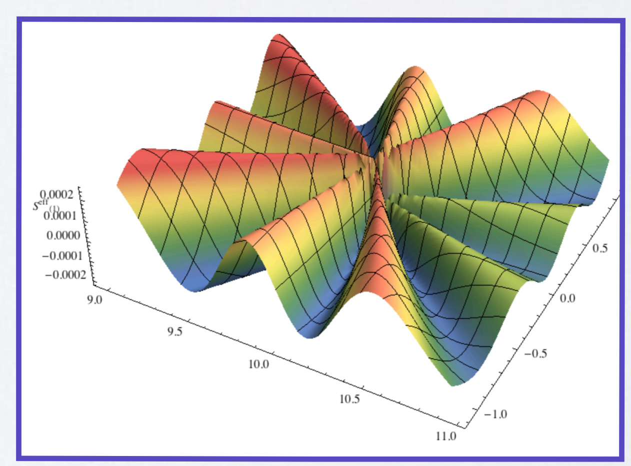
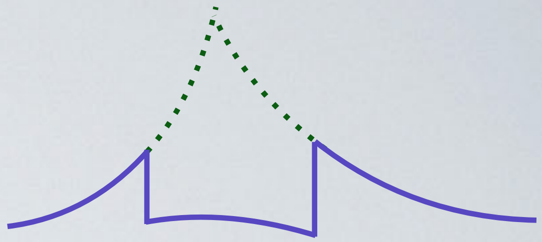


2. Mode-sum regularization

3. Effective source methods



Worldline convolution was the first method of regularization proposed



- $\rho(x^\mu)$

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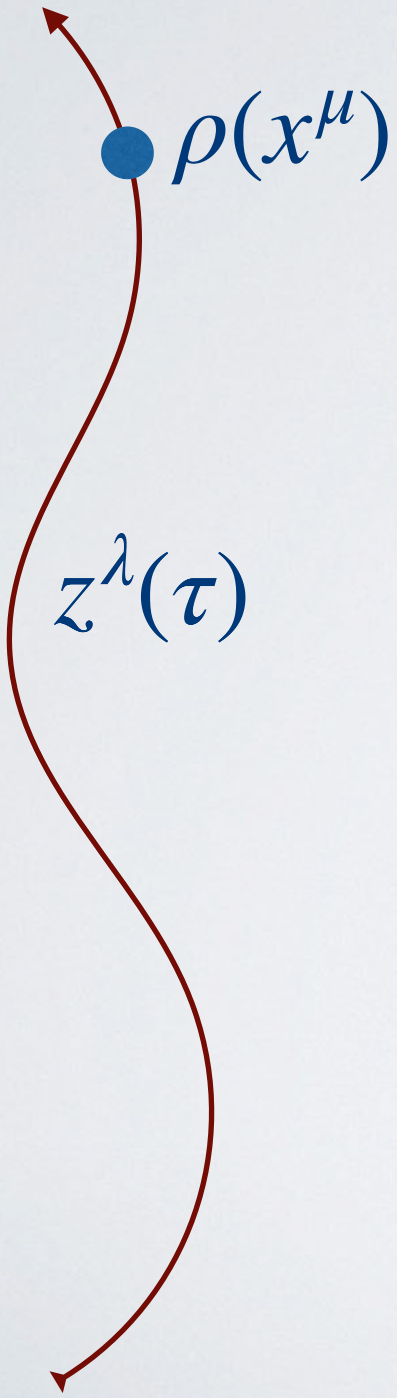

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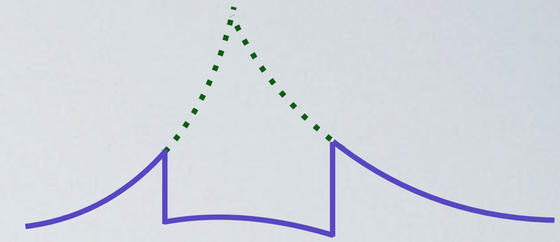
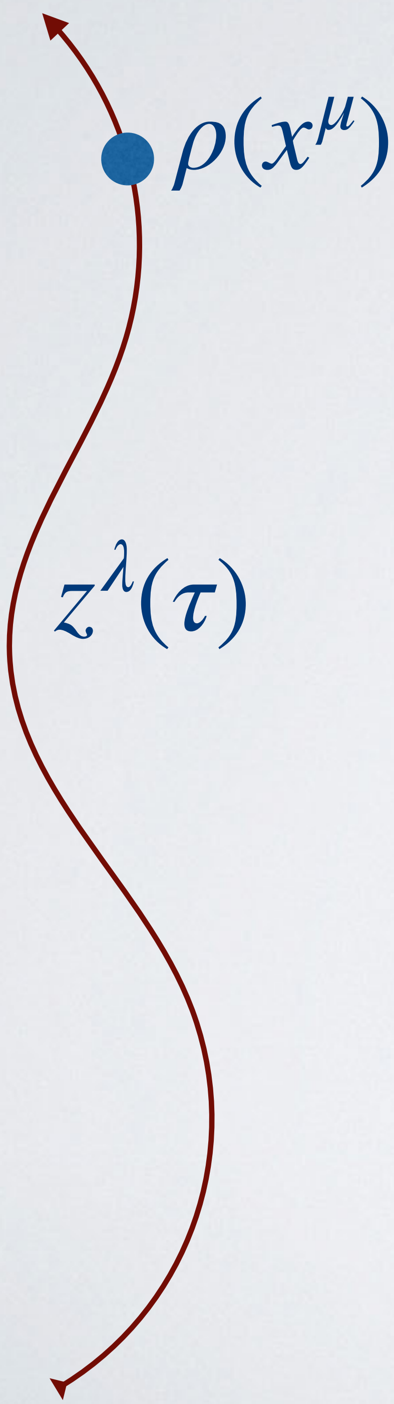
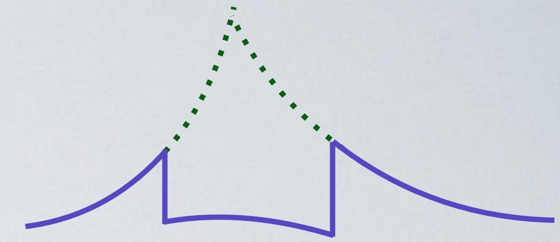
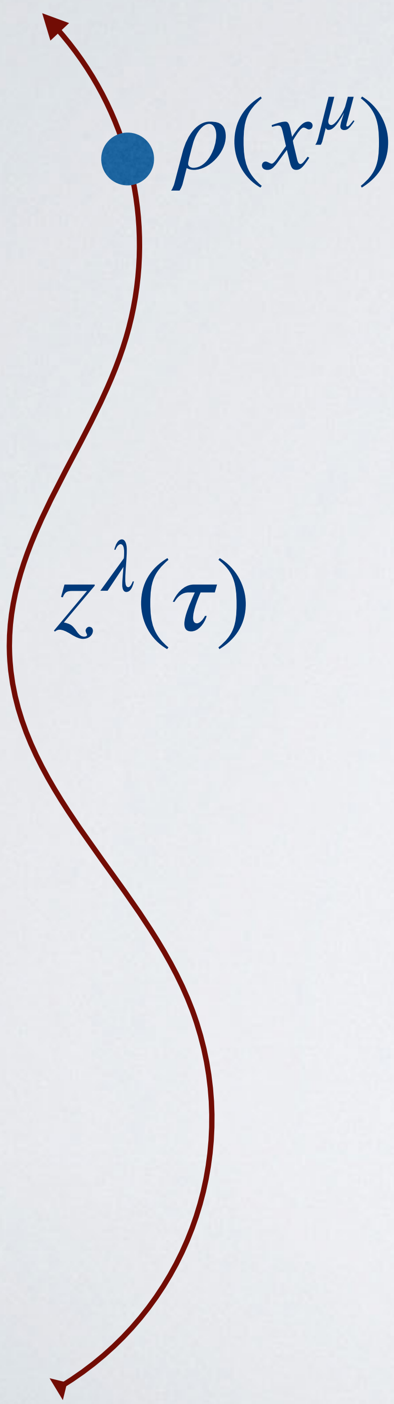




$$\Box\Psi(x^\mu) = 4\pi\rho(x^\mu)$$

Worldline convolution was the first method of regularization proposed





$$\Box\Psi(x^\mu) = 4\pi\rho(x^\mu)$$

$$\Psi(x^\mu) = \int G_{\text{ret}}(x^\mu, x'^\mu)\rho(x'^\mu)d^4x'$$

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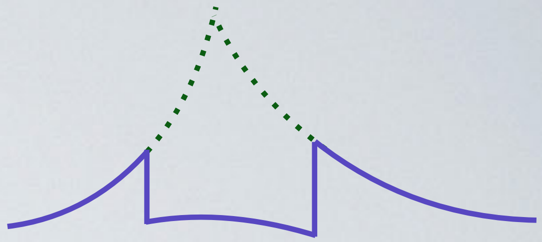

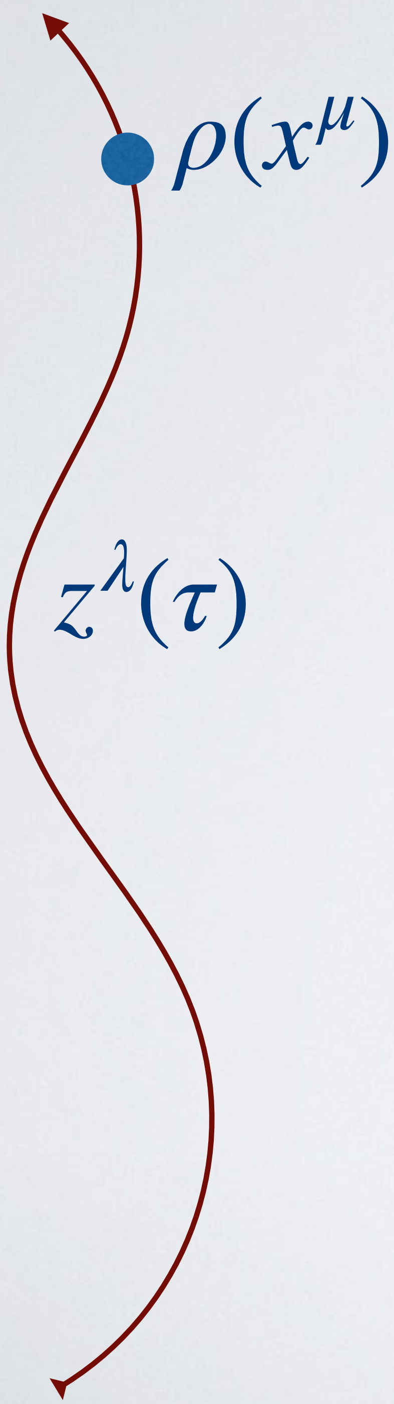

$$\Box \Psi(x^\mu) = 4\pi\rho(x^\mu) \qquad \Psi(x^\mu) = \int G_{\text{ret}}(x^\mu, x'^\mu)\rho(x'^\mu) d^4x'$$

1. $\Box G_{\text{ret}}(x^\mu, x'^\mu) = 4\pi\delta(x^\mu, x'^\mu)$

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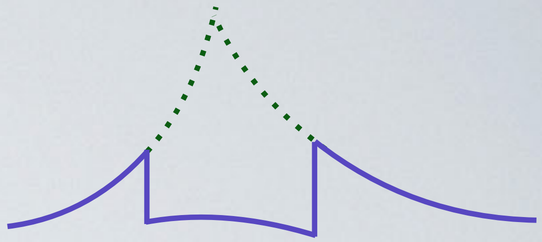
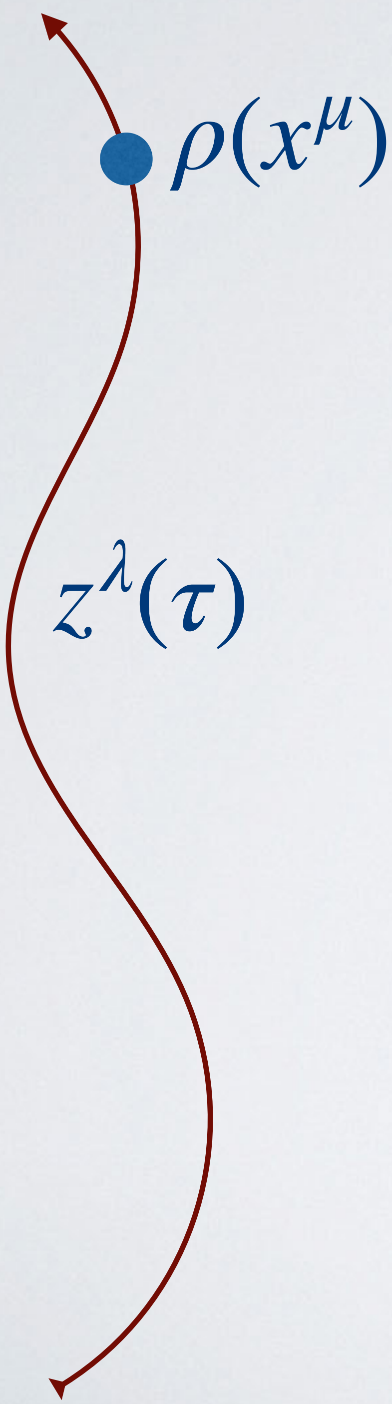


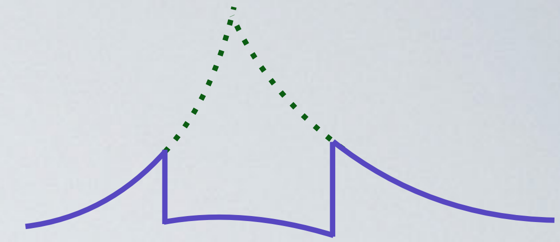


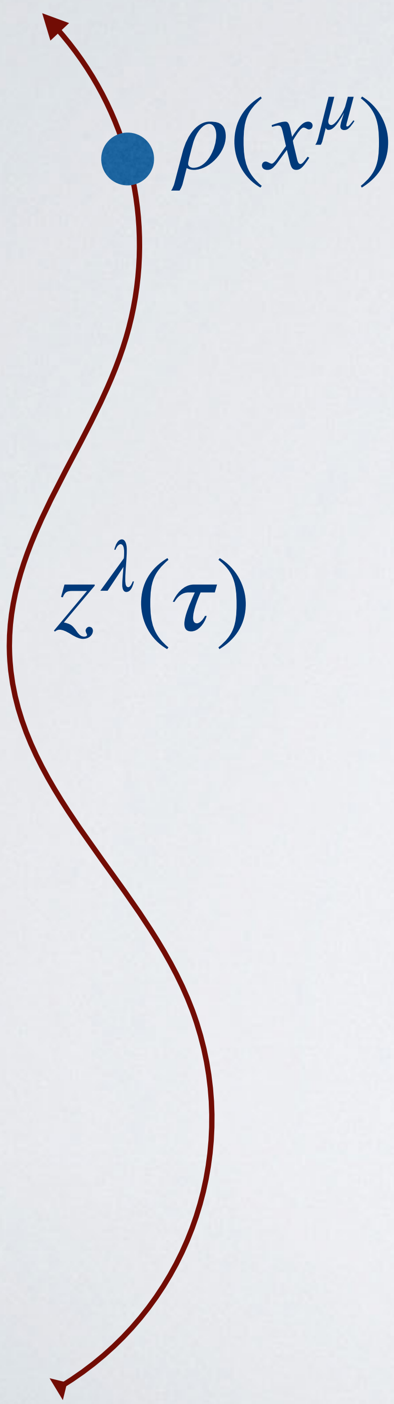
$$\Box\,\Psi(x^\mu) = 4\pi\rho(x^\mu) \qquad \Psi(x^\mu) = \int G_{\text{ret}}(x^\mu, x'^\mu)\rho(x'^\mu)d^4x'$$

1. $$\Box\, G_{\text{ret}}(x^\mu, x'^\mu) = 4\pi\delta(x^\mu, x'^\mu)$$

2. $$F^\alpha[z^\lambda] = (\text{local terms}) + \lim_{\epsilon\to 0}\int_{-\infty}^{\tau-\epsilon} \nabla^\alpha G_{\text{ret}}(z^\lambda, z'^\lambda)d\tau'$$

Worldline convolution was the last method of regularization implemented



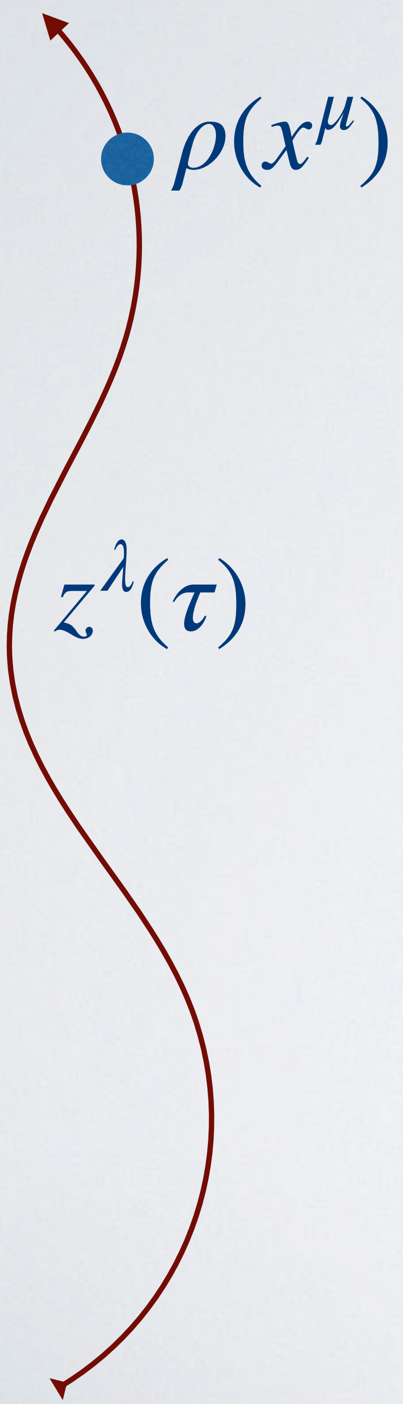


Self-force via Green functions and worldline integration

Barry Wardell,[1,2] Chad R. Galley,[3] Anıl Zenginoğlu,[3] Marc Casals,[4] Sam R. Dolan,[5] and Adrian C. Ottewill[1]

[1]*School of Mathematical Sciences and Complex & Adaptive Systems Laboratory, University College Dublin, Belfield, Dublin 4, Ireland*
[2]*Department of Astronomy, Cornell University, Ithaca, NY 14853, USA*
[3]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California USA*
[4]*Department of Cosmology, Relativity and Astrophysics (ICRA), Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, CEP 22290-180, Brazil.*
[5]*Consortium for Fundamental Physics, School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom.*

Worldline convolution was the last method of regularization implemented



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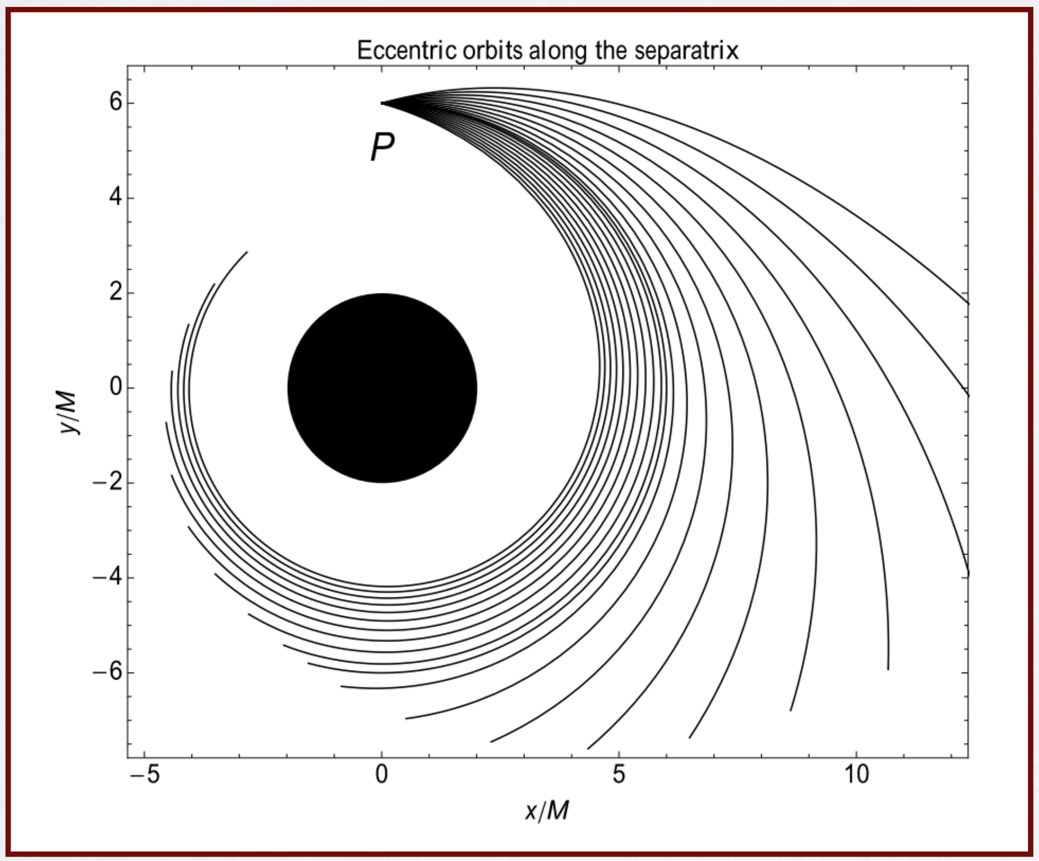
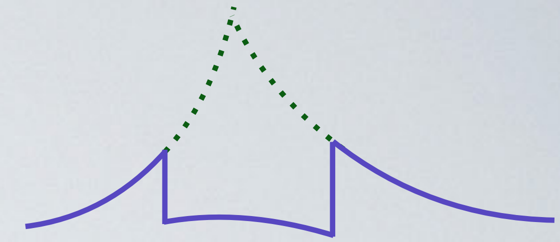
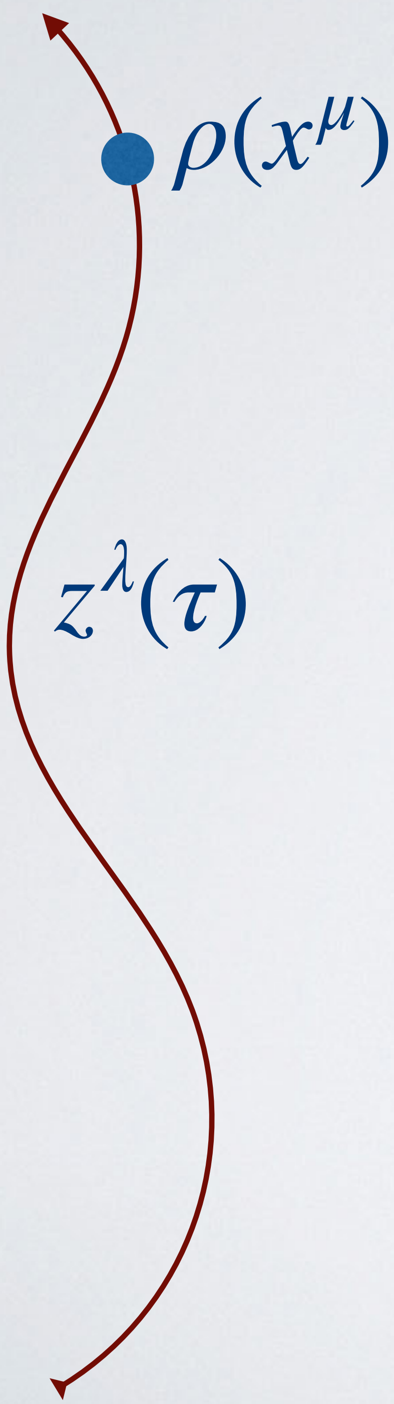


Worldline convolution was the last method of regularization implemented





Self-force via Green functions and worldline integration

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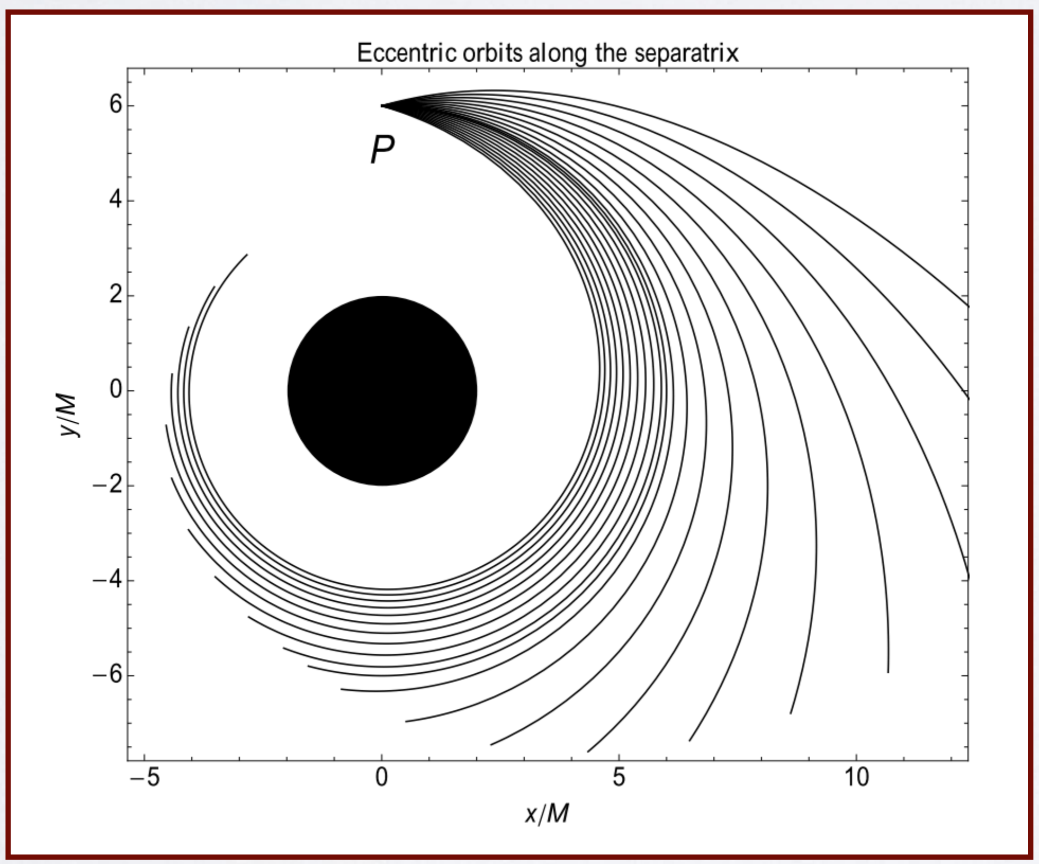
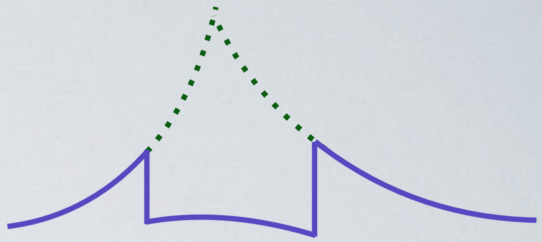
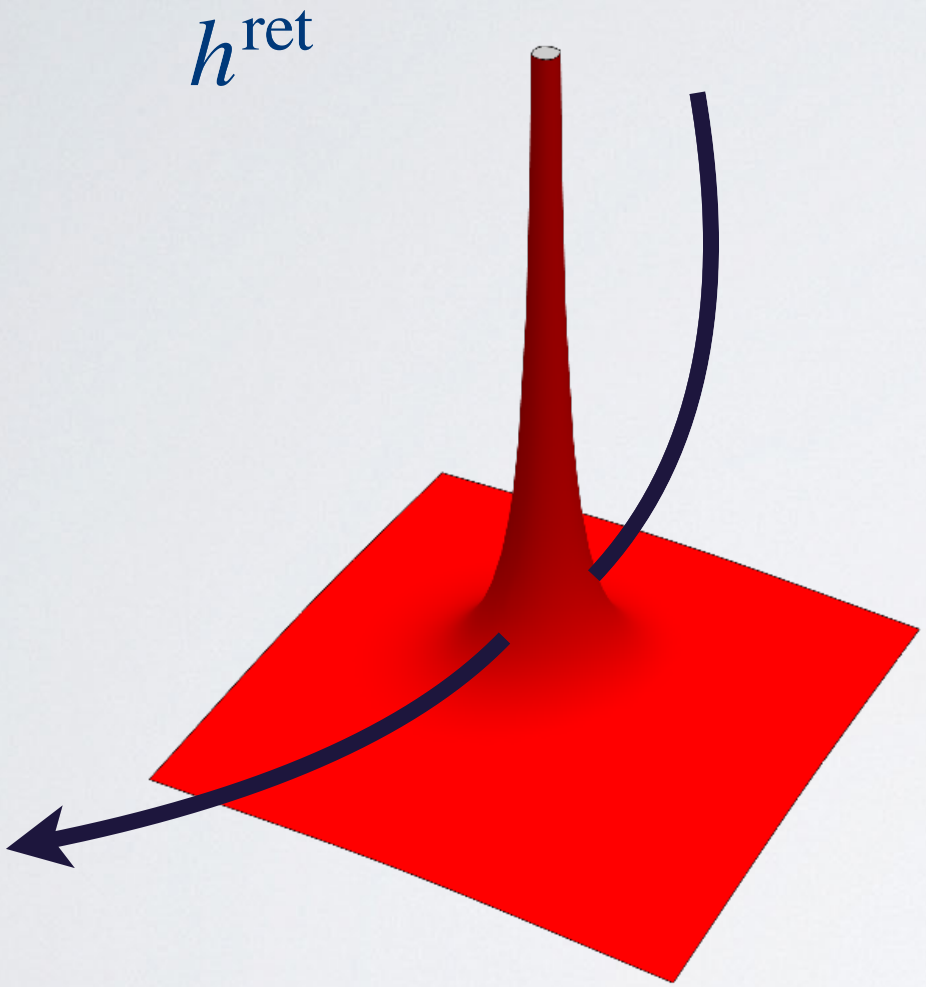


Self-force available for *any worldline* passing through *P*

In mode-sum regularization the finite contribution to the 1/r field is subtracted mode-by-mode





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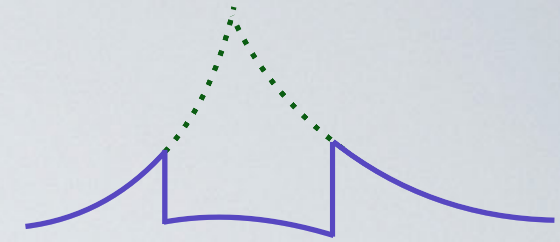

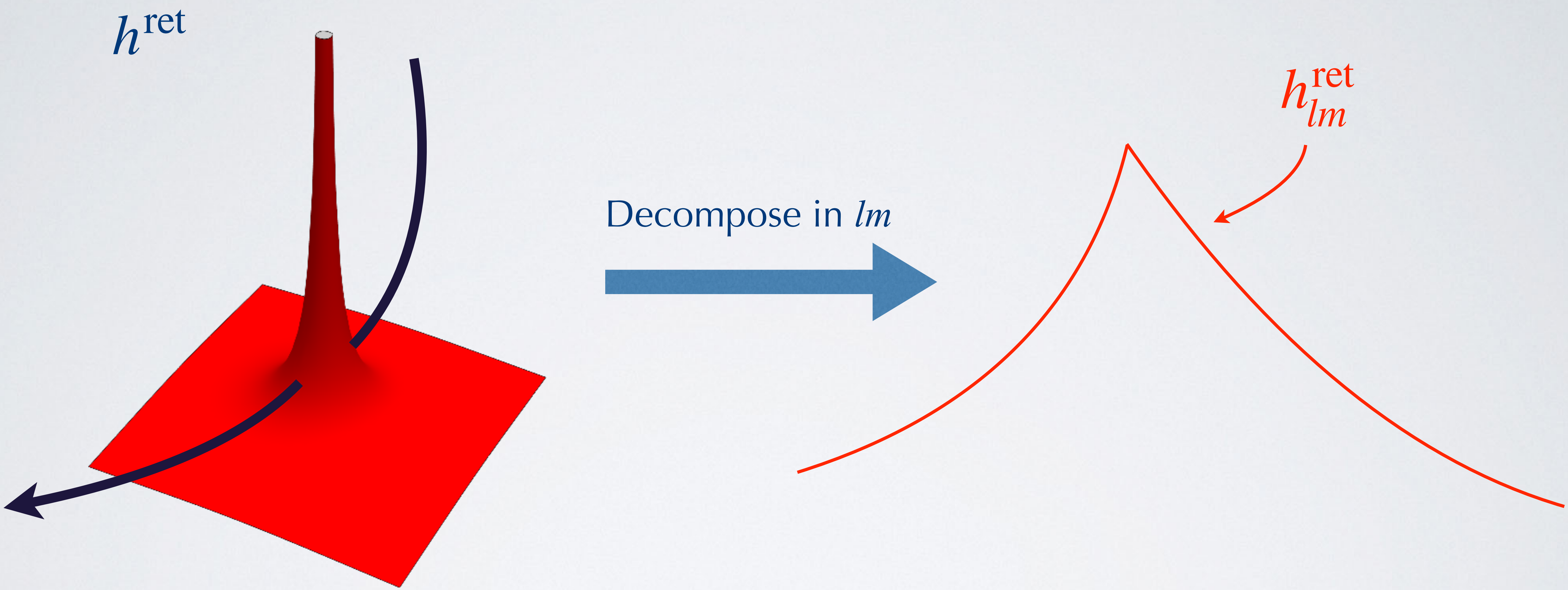

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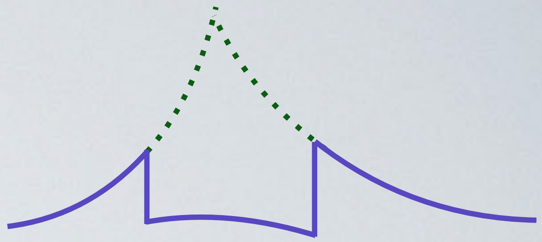
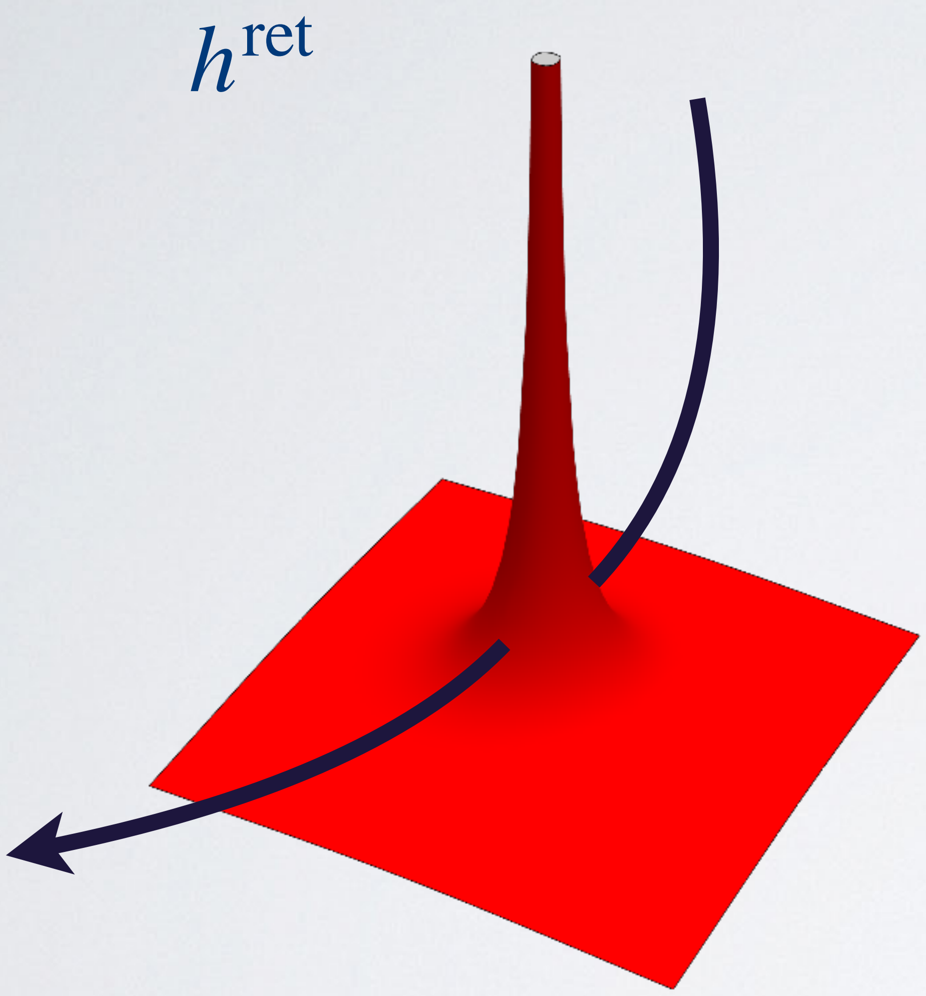
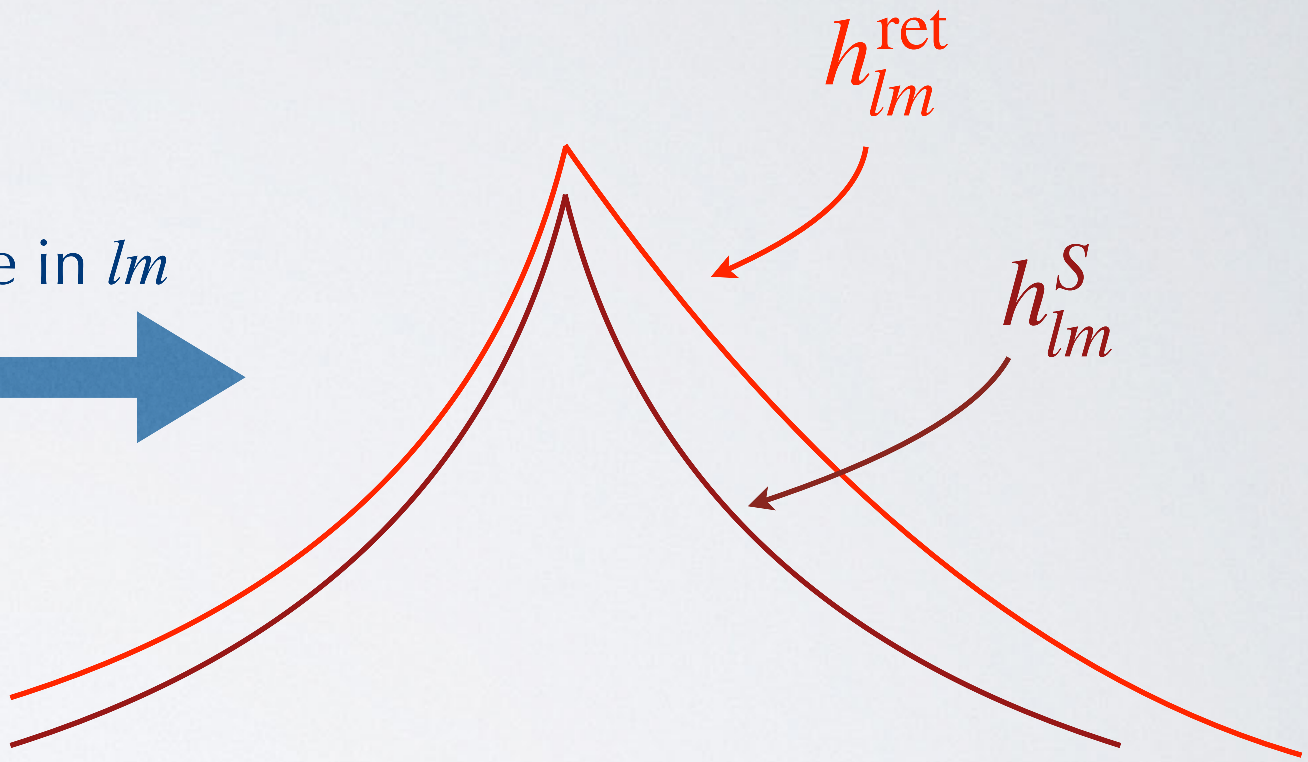
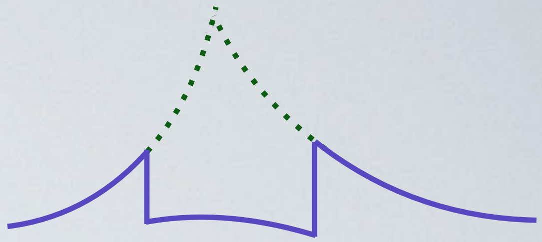


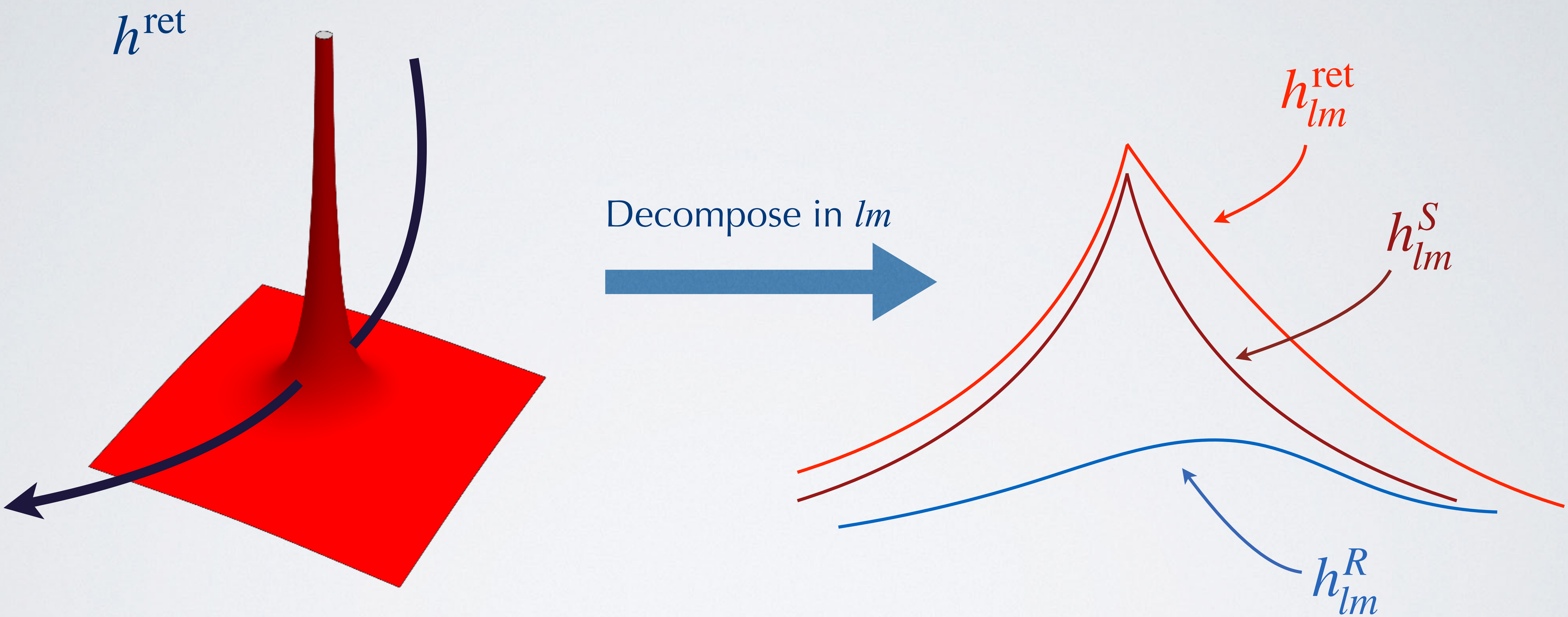


Decompose in *lm*



In mode-sum regularization the finite contribution to the 1/r field is subtracted mode-by-mode





Mode-sum regularization was the first scheme used to compute the GSF

Mode sum regularization approach for the self-force in black hole spacetime

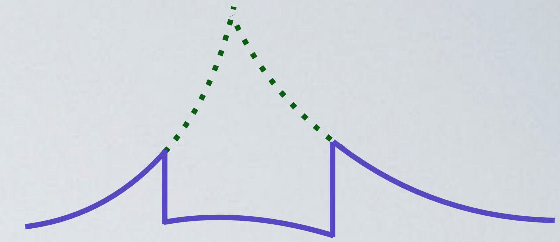
Leor Barack and Amos Ori
Department of Physics, Technion—Israel Institute of Technology, Haifa, 32000, Israel
(December 5, 1999)

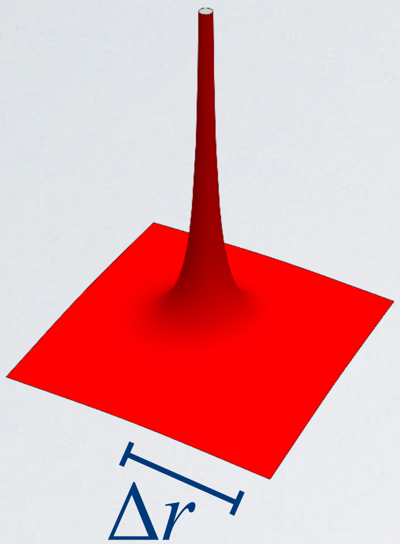
We present a method for calculating the self-force (the "radiation reaction force") acting on a charged particle moving in a strong field orbit in black hole spacetime. In this approach, one first calculates the contribution to the self-force due to each multipole mode of the particle's field. Then, the sum over modes is evaluated, subject to a certain regularization procedure. Here we develop this regularization procedure for a scalar charge on a Schwarzschild background, and present the results of its implementation for radial trajectories (not necessarily geodesic).

Calculating the gravitational self force in Schwarzschild spacetime

Leor Barack[1], Yasushi Mino[2], Hiroyuki Nakano[3], Amos Ori[4], and Misao Sasaki[3]
[1] *Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1, D-14476 Golm, Germany*
[2] *Theoretical Astrophysics, California institute of Technology, Pasadena, California 91125*
[3] *Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[4] *Department of Physics, Technion—Israel Institute of Technology, Haifa, 32000, Israel*
(May 29, 2001)

The singular field is found analytically at the particle and removed *l*-by-*l*





$\phi^S(x^\mu)$ known as expansion in Δr

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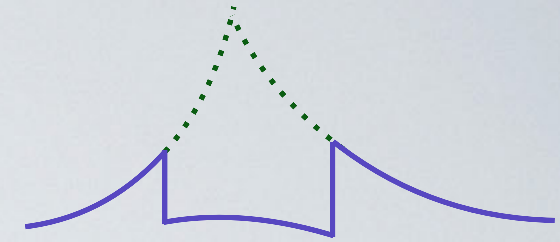
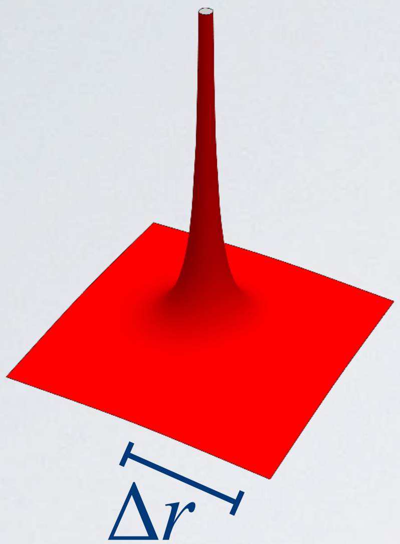
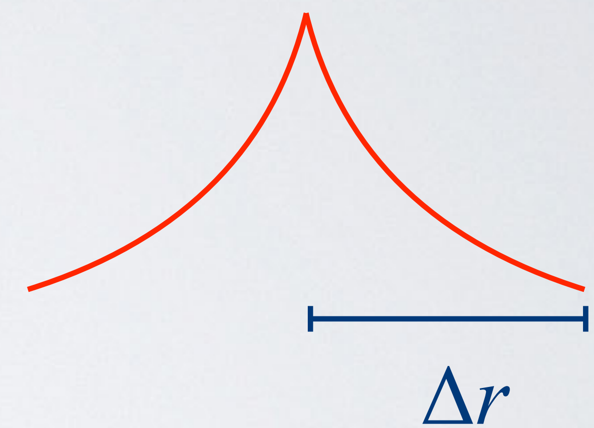




$\phi^S(x^\mu)$ known as expansion in Δr

$$\phi^S_{lm}(t,r) = \int \phi^S(x^\mu) Y^*_{lm}(\theta,\varphi)\, d\Omega$$



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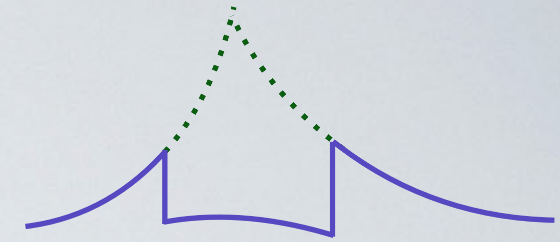

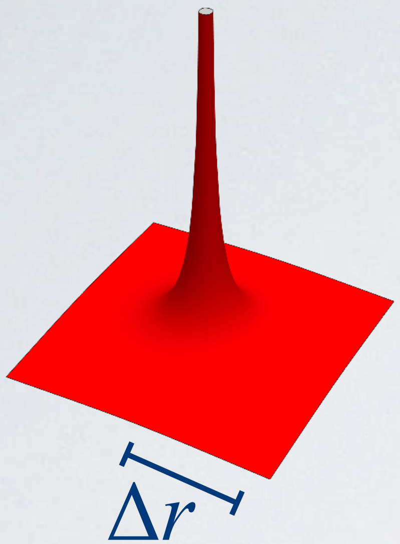

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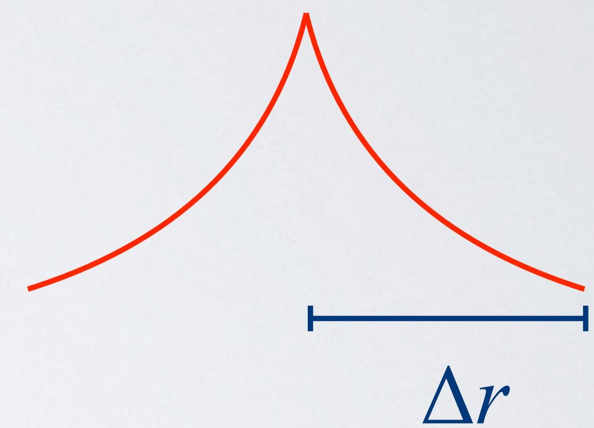
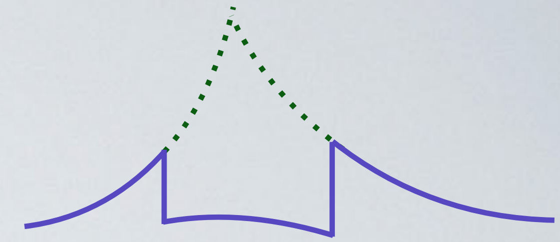
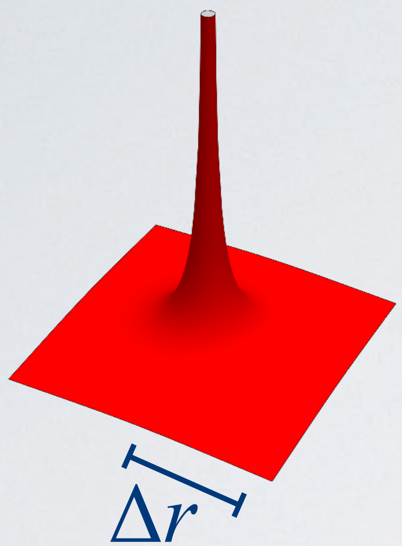


$$F^\mu = \sum_{l=0}^{\infty} F_l^{\mathrm{ret}\,\mu} - A_l\left(l + 1/2\right) - B_l - \cdots$$

The singular field is found analytically at the particle and removed *l*-by-*l*





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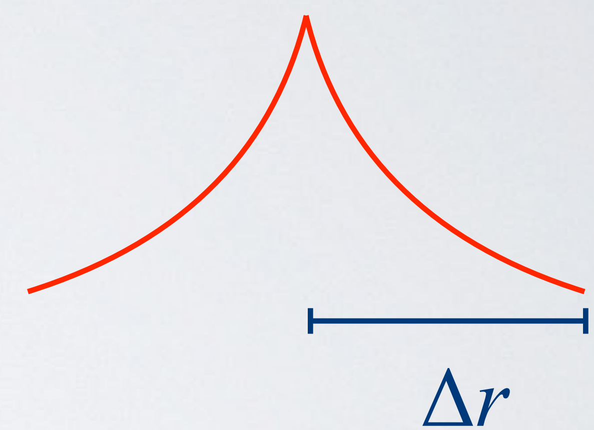

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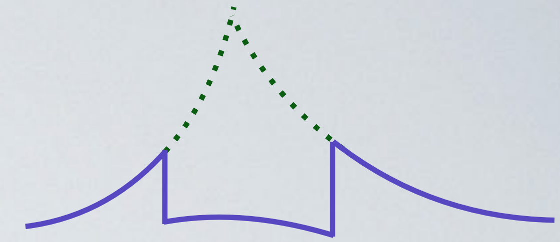

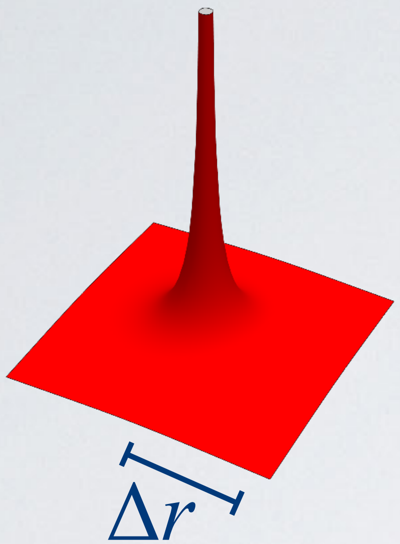

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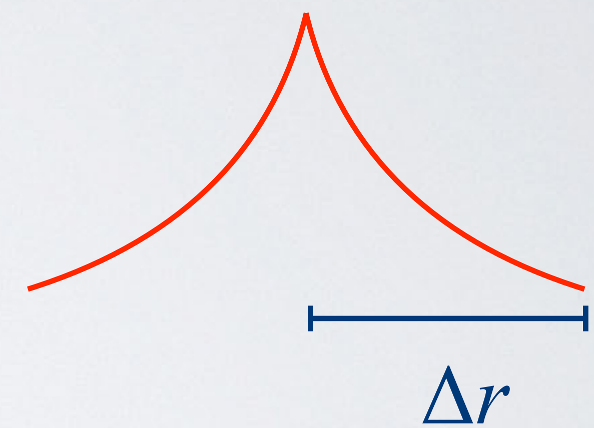

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A_l, B_l: Regularization parameters

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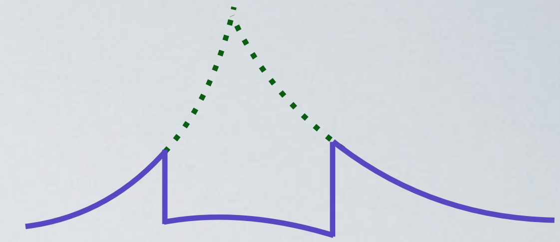

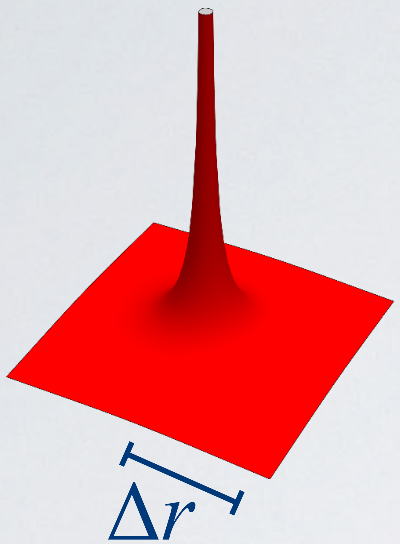

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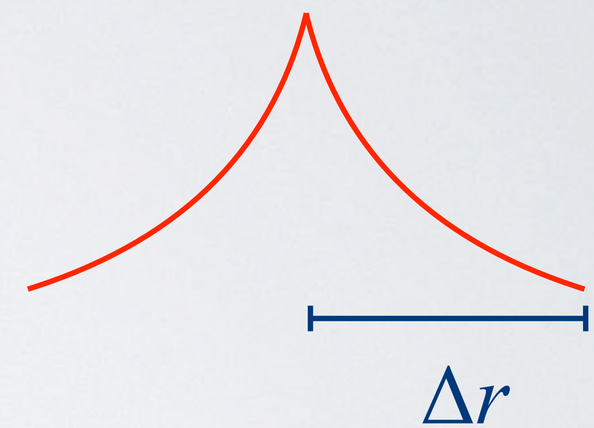

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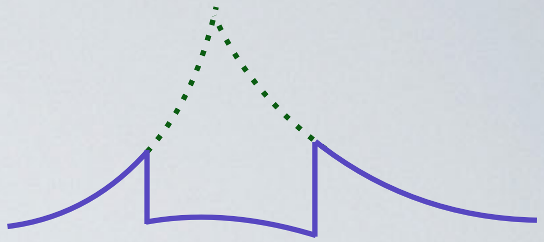
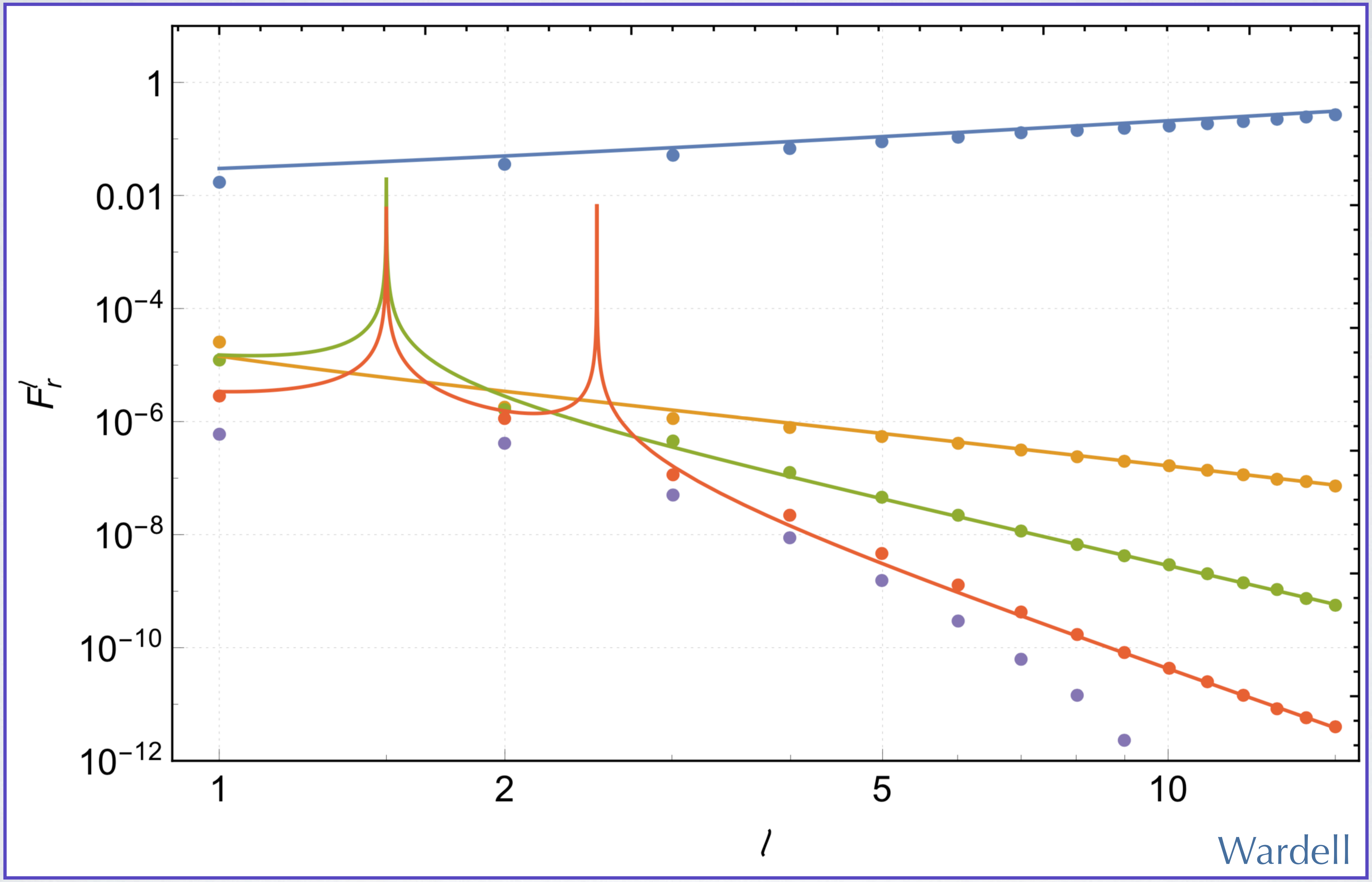
A_l, B_l: Regularization parameters

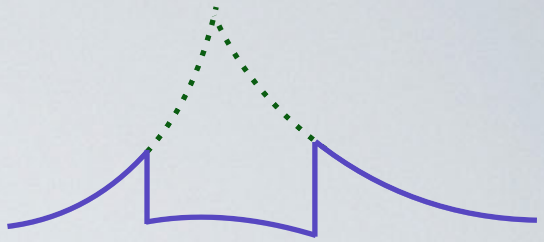
Anna Heffernan - Tuesday

Higher-order singular fields produce faster convergence in l





Effective source calculations are the most recent schemes implemented



$$\Box \Psi^{\rm ret} = 4\pi\rho$$

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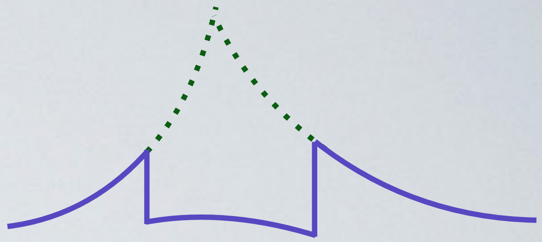

$$\Box\,\Psi^{\rm ret} = 4\pi\rho \qquad\qquad \Box\,\Psi^{R} = \Box\,\Psi^{\rm ret} - \Box\Psi^{S}$$

Effective source calculations are the most recent schemes implemented

$$\Box \Psi^{\mathrm{ret}} = 4\pi\rho$$

$$\begin{aligned} \Box \Psi^{R} &= \Box \Psi^{\mathrm{ret}} - \Box \Psi^{S} \\ &= 4\pi\rho - \Box \Psi^{S} \end{aligned}$$

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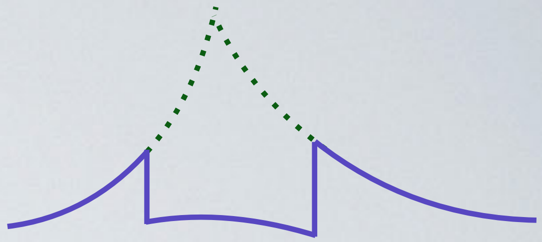

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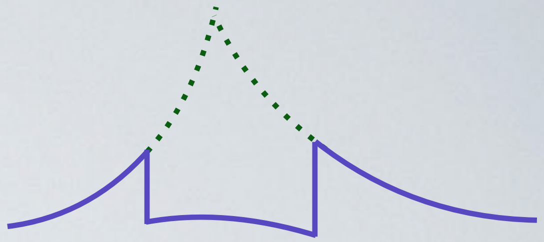

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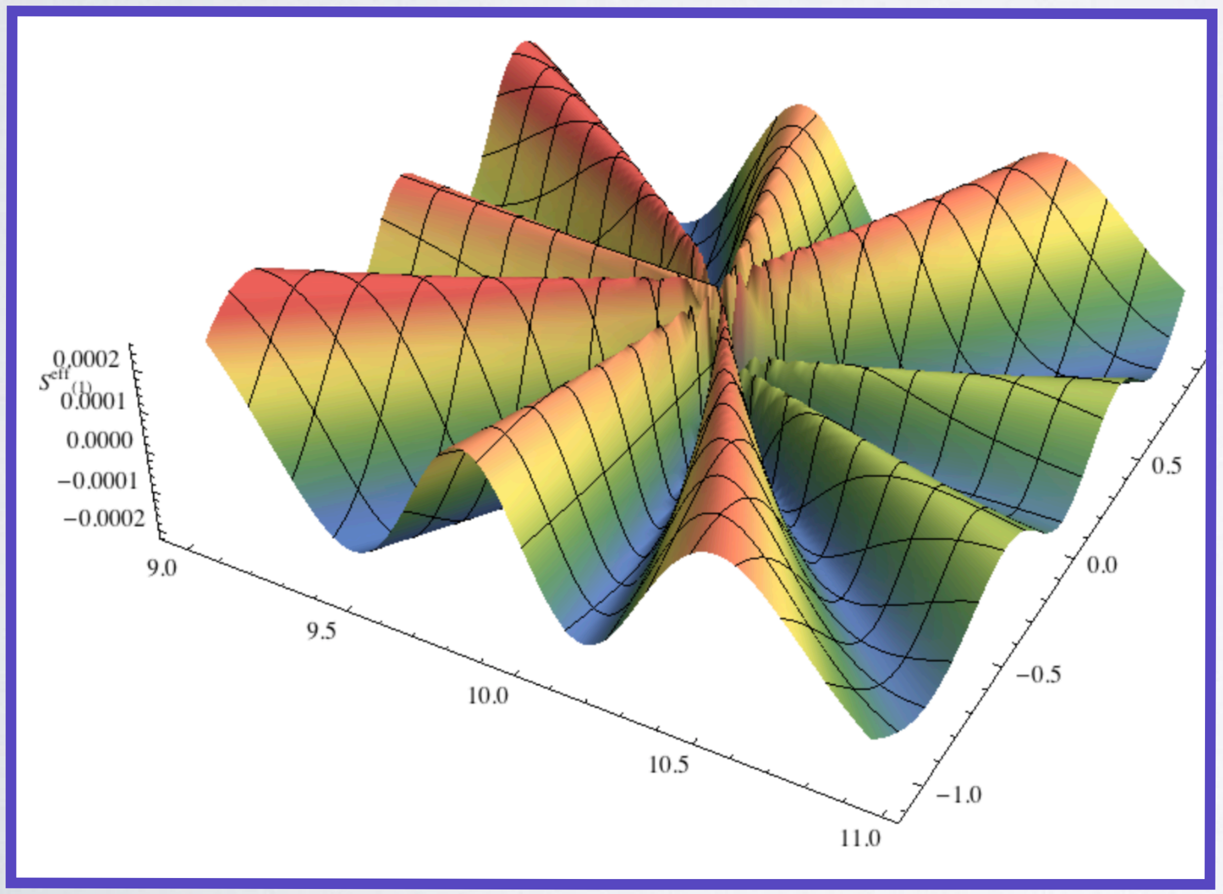
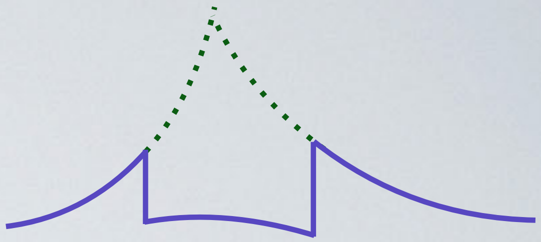


Wardell

Effective source calculations are the most recent schemes implemented



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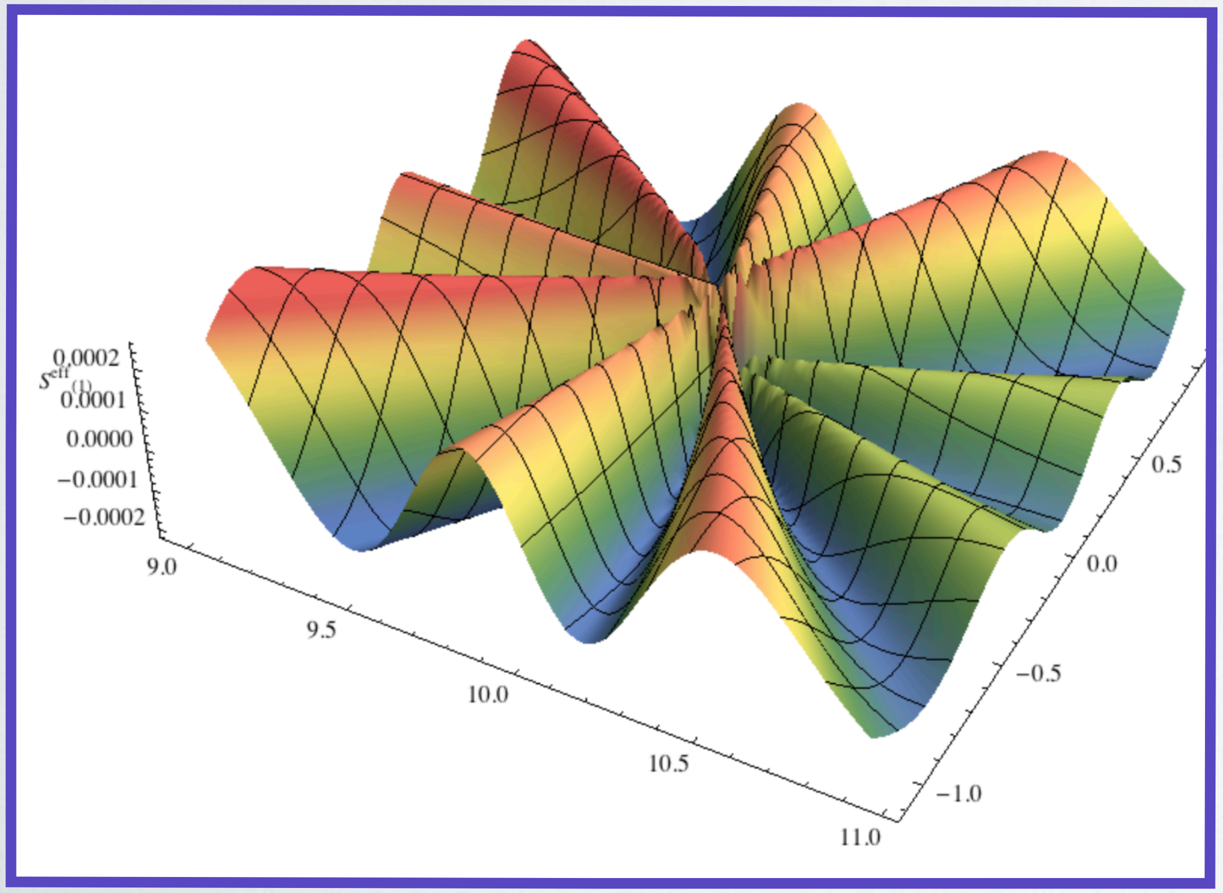
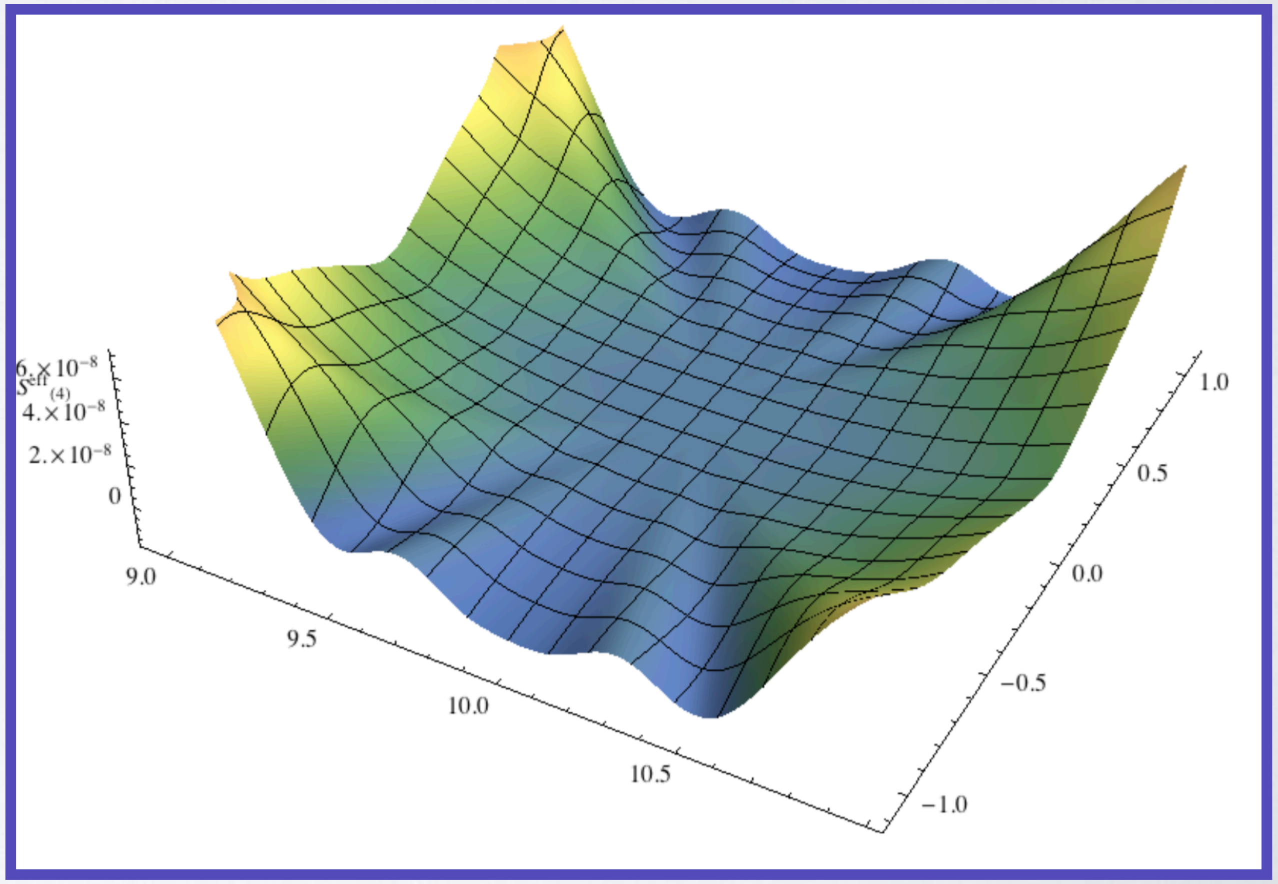
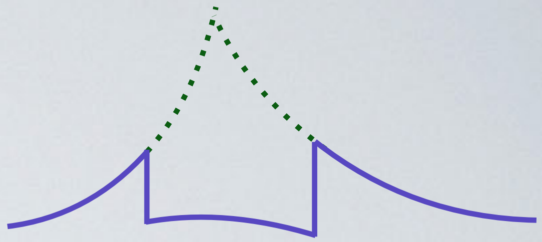




Wardell

Effective source calculations are the most recent schemes proposed



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Scalar-field perturbations from a particle orbiting a black hole using numerical evolution in 2 + 1 dimensions

Leor Barack and Darren A. Golbourn
School of Mathematics, University of Southampton, Southampton, SO17 1BJ, United Kingdom
(Received 24 May 2007; published 23 August 2007)

We present a new technique for time-domain numerical evolution of the scalar- field generated by a pointlike scalar charge orbiting a black hole. Time-domain evolution offers an efficient way for calculating black hole perturbations, especially as input for computations of the local self force acting on orbiting particles. In Kerr geometry, the field equations are not fully separable in the time domain, and one has to tackle them in 2 + 1 dimensions (two spatial dimensions and time; the azimuthal dependence is

Regularization of fields for self-force problems in curved spacetime: foundations and a time-domain application

Ian Vega and Steven Detweiler
Institute for Fundamental Theory, Department of Physics,
University of Florida, Gainesville, FL 32611-8440[*]
(Dated: January 15, 2008)

We propose an approach for the calculation of self-forces, energy fluxes and waveforms arising from moving point charges in curved spacetimes. As opposed to mode-sum schemes that regularize the self-force derived from the singular retarded field, this approach regularizes the retarded field itself. The singular part of the retarded field is first analytically identified and removed, yielding a finite, differentiable remainder from which the self-force is easily calculated. This regular remainder

Effective source codes are well-suited to self-consistent evolutions

$$\Box \Psi^R = 4\pi\rho_{\text{eff}} \qquad\qquad \frac{D^2 z^\mu}{d\tau^2} = \frac{q}{m}(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \Psi^R$$

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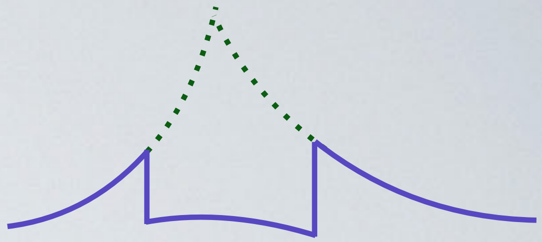

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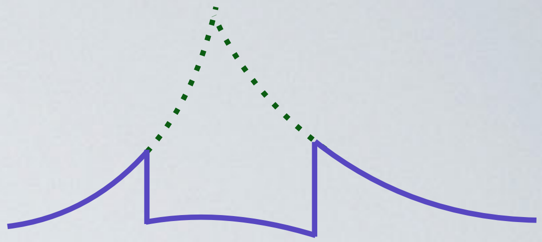

$$\Box\, \Psi^R = 4\pi\rho_{\rm eff} \qquad \frac{D^2 z^\mu}{d\tau^2} = \frac{q}{m}(g^{\mu\nu} + u^\mu u^\nu)\,\nabla_\nu \Psi^R$$

The Good

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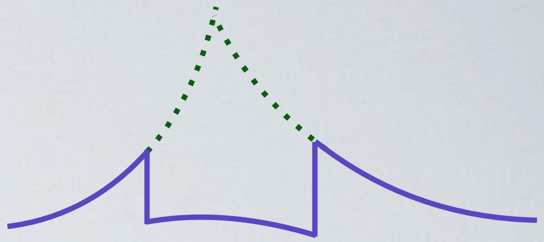

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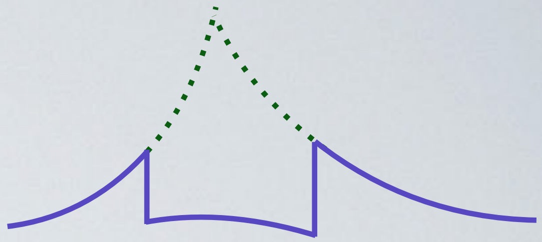

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- No singular fields
- Generic trajectories

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The Good

- No singular fields
- Generic trajectories
- Works at second order

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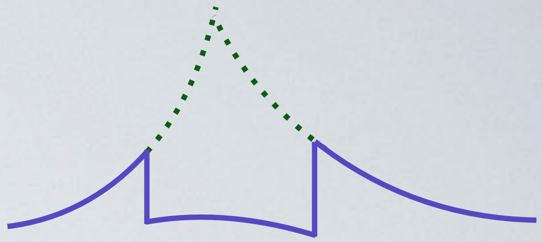

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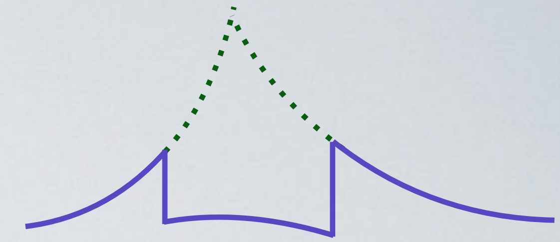

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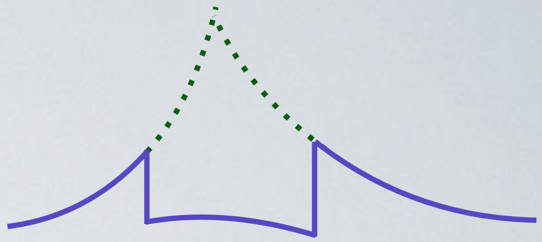
- Computationally expensive

Effective source codes are well-suited to self-consistent evolutions



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The Good

- No singular fields
- Generic trajectories
- Works at second order

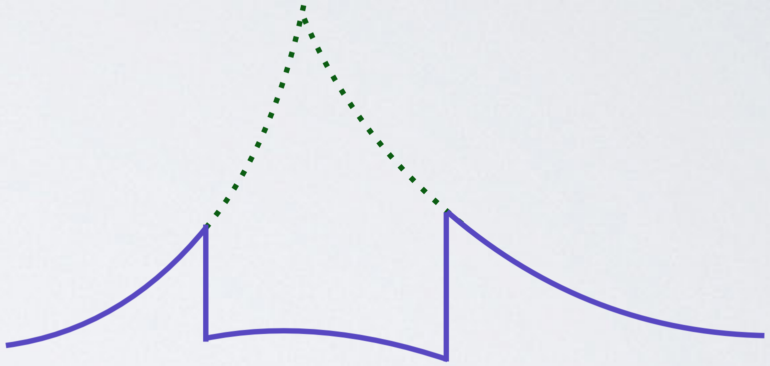
The Bad

- Computationally expensive
- Effective source is cumbersome

Yesterday

History

Regularization



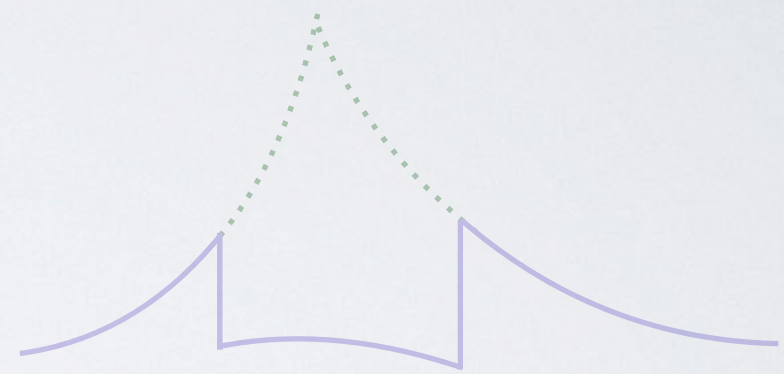
Practical considerations

Gauge invariants

Yesterday

History

Regularization

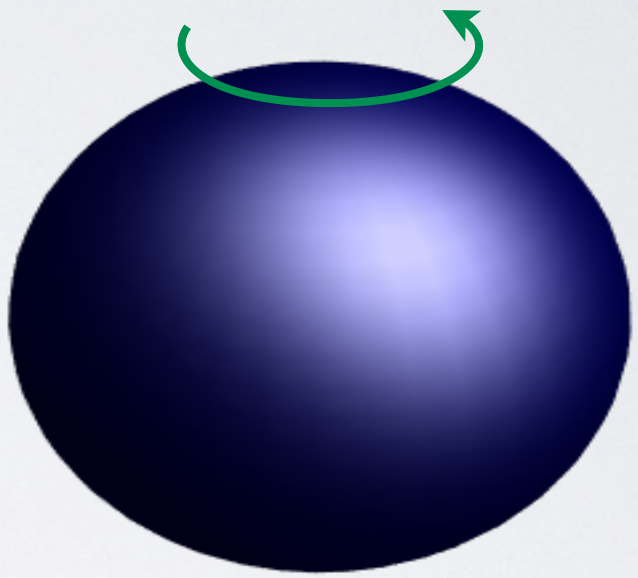


Practical considerations

Gauge invariants

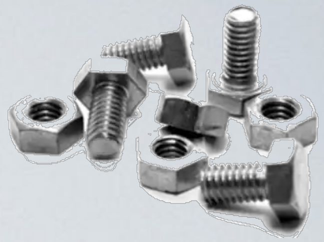
We have a choice of background spacetime

Schwarzschild



Kerr

There are many methods for solving the differential equations



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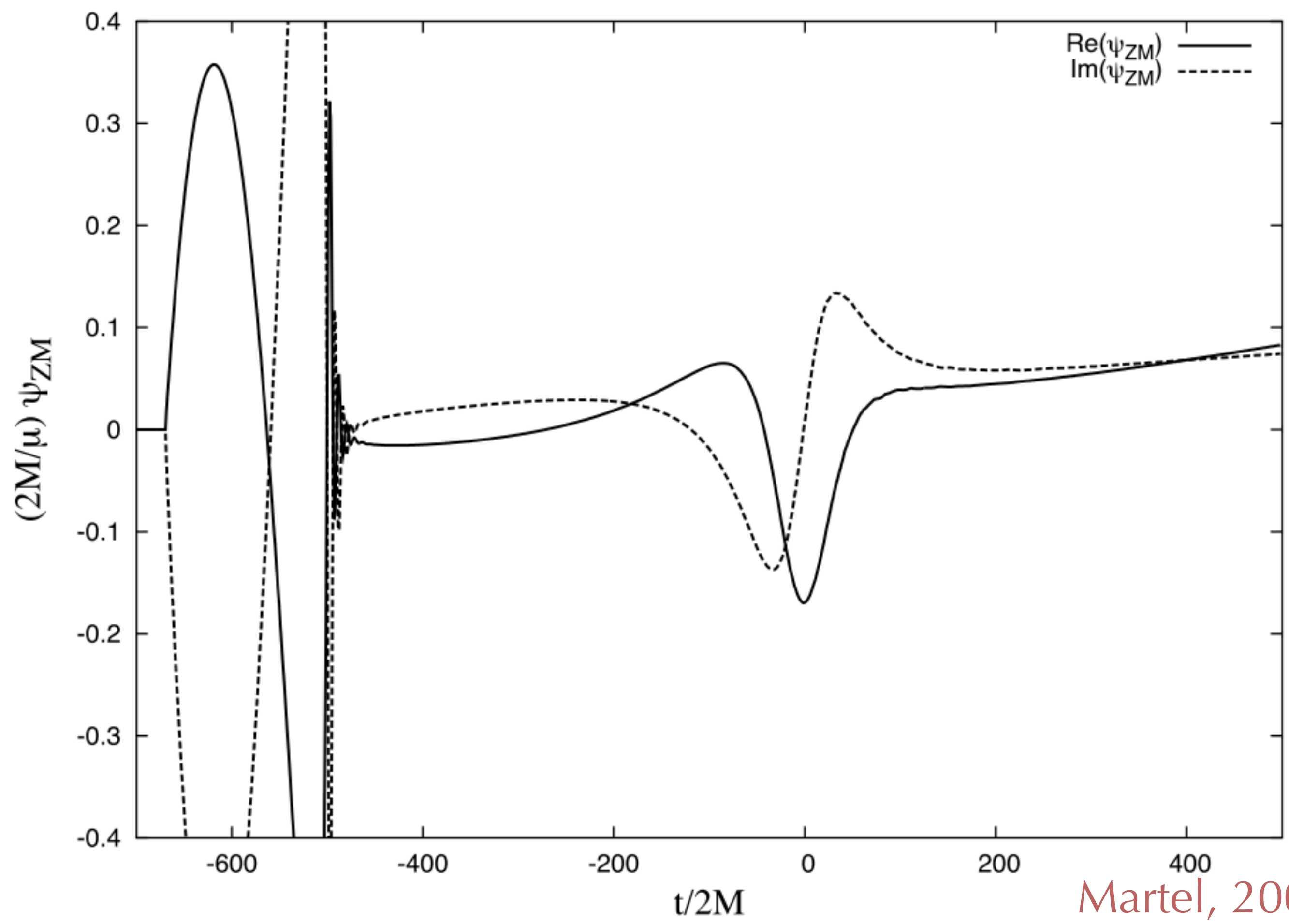
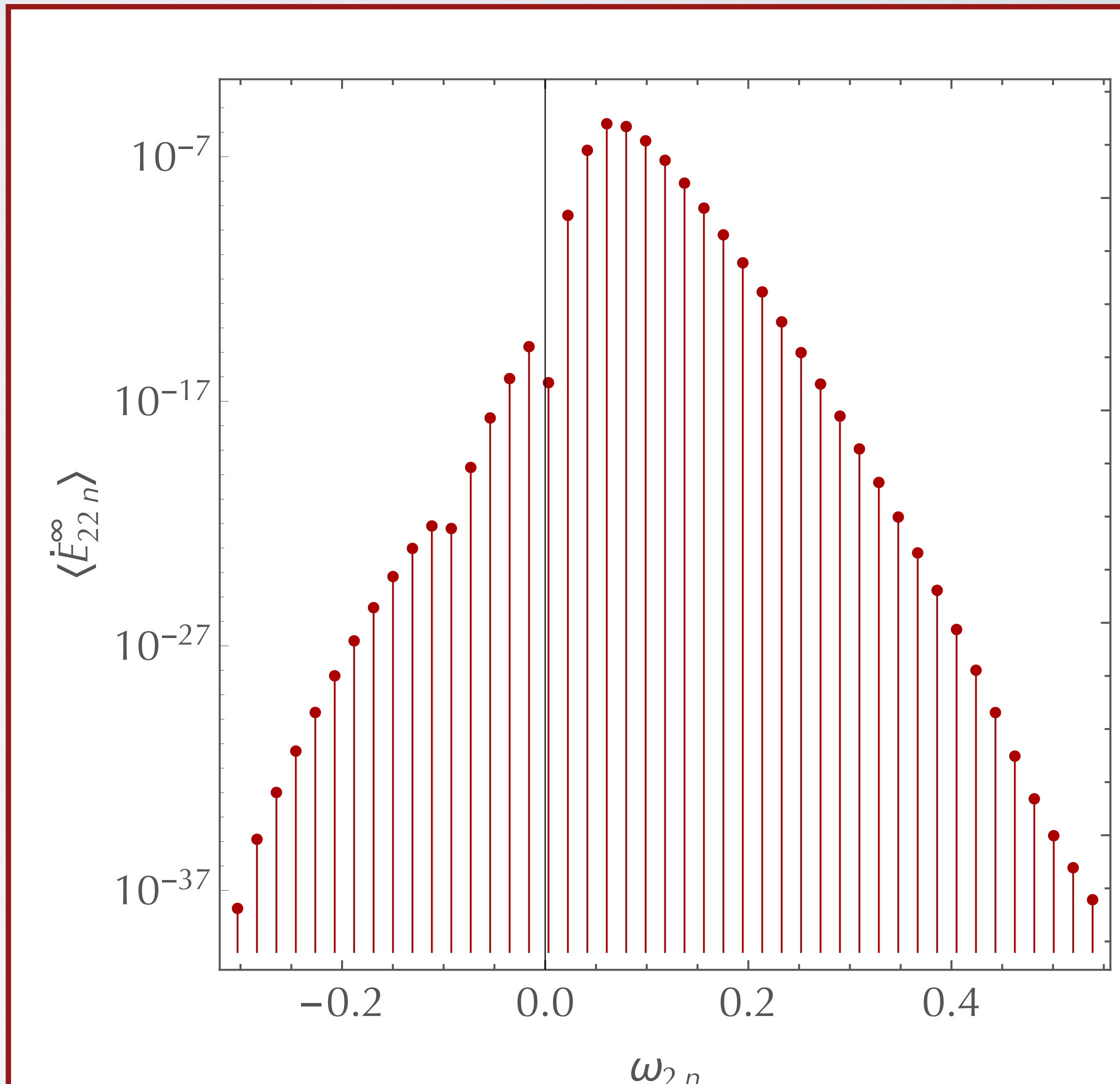


Martel, 2003

Frequency domain



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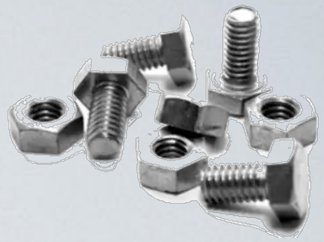

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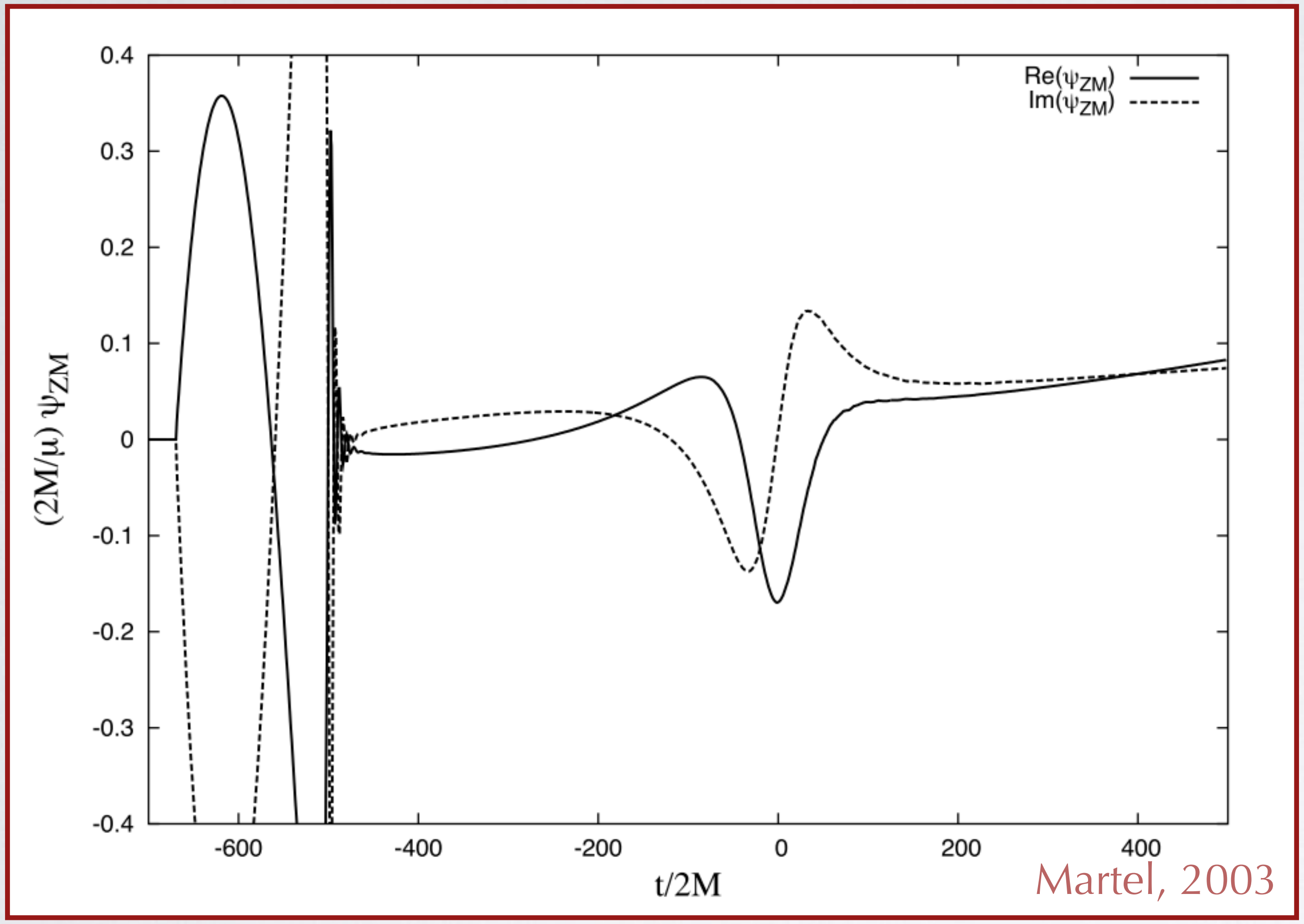

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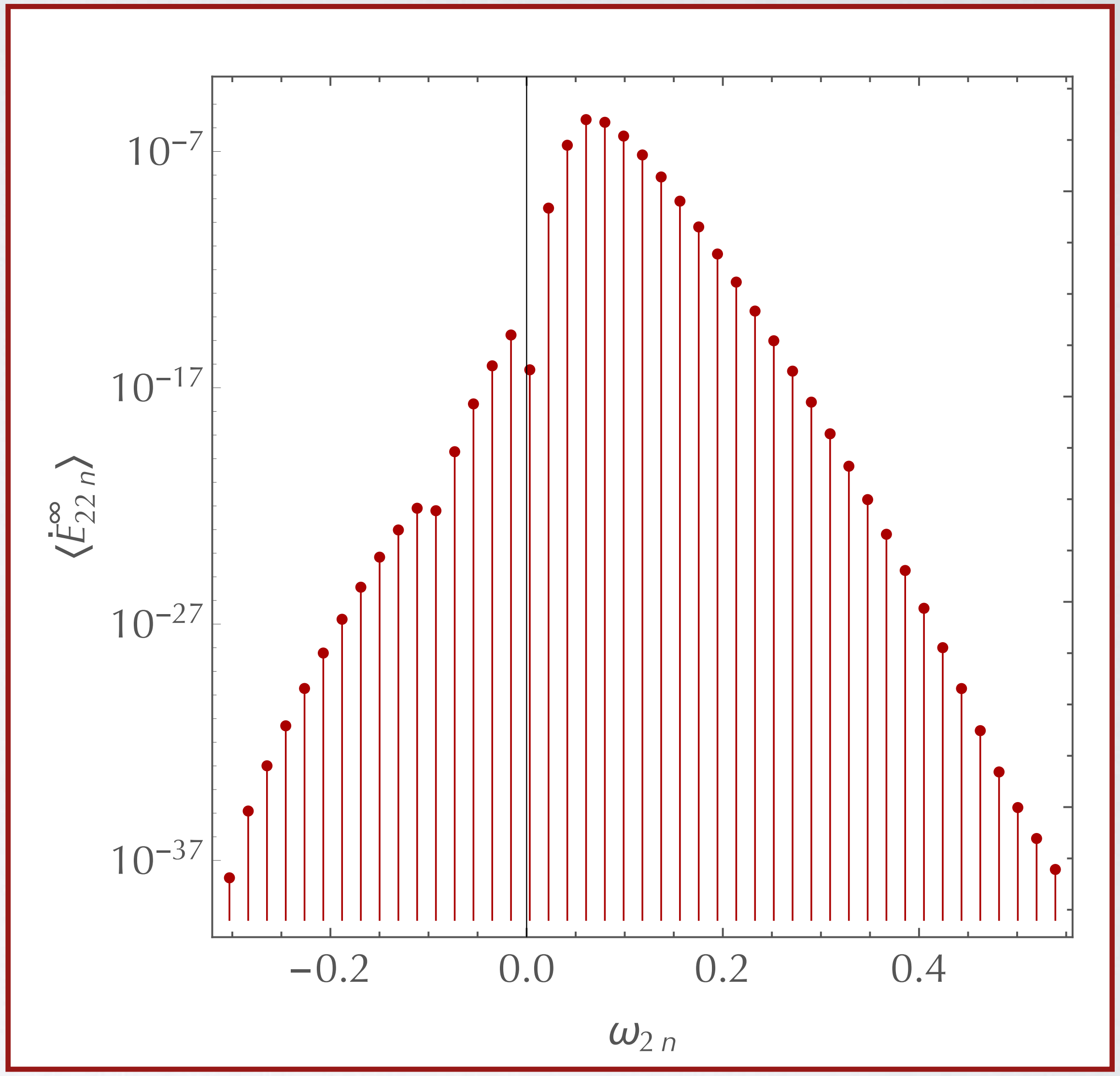
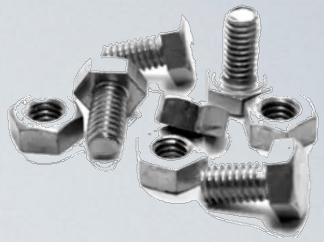


- Finite difference

There are many methods for solving the differential equations



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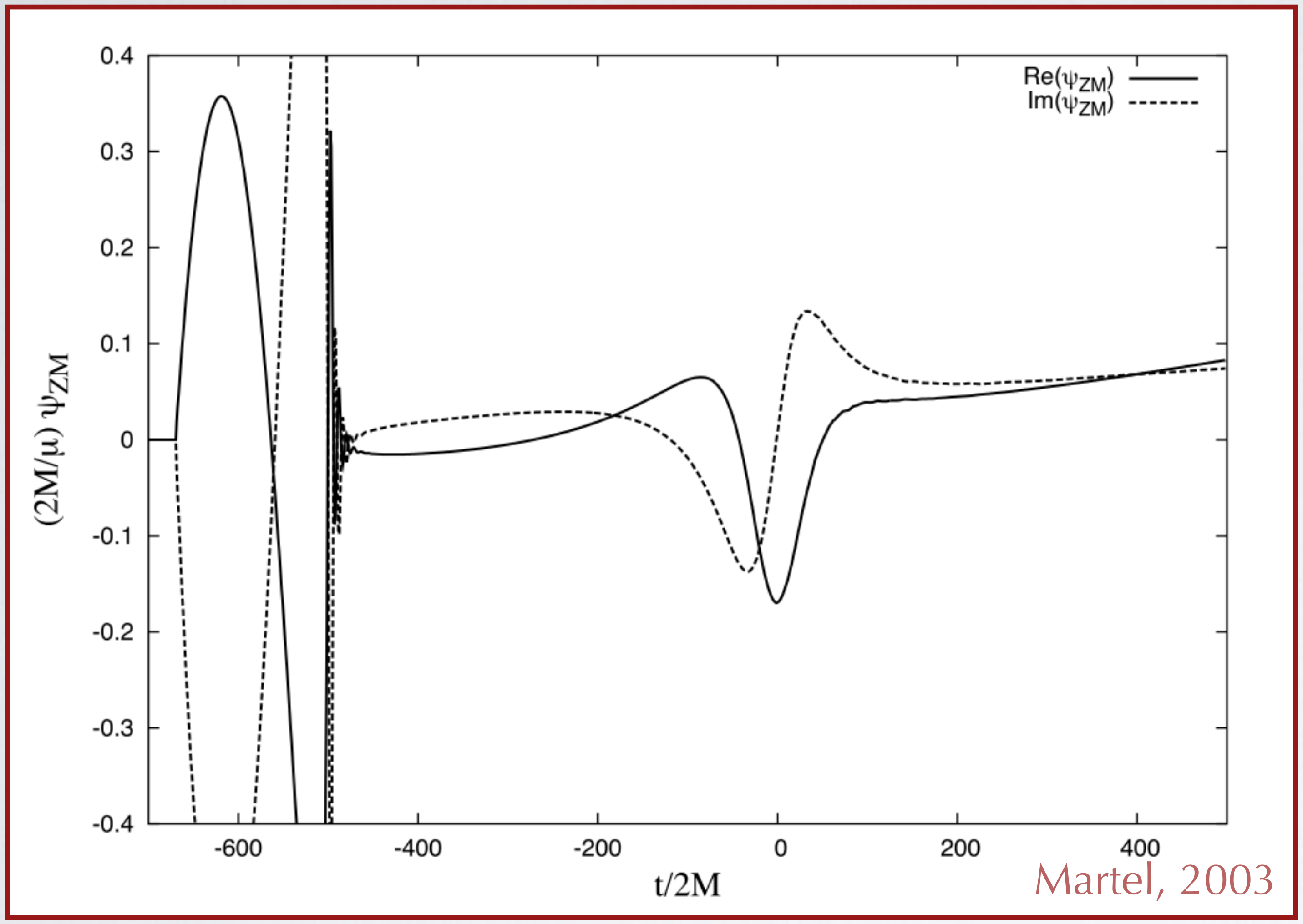

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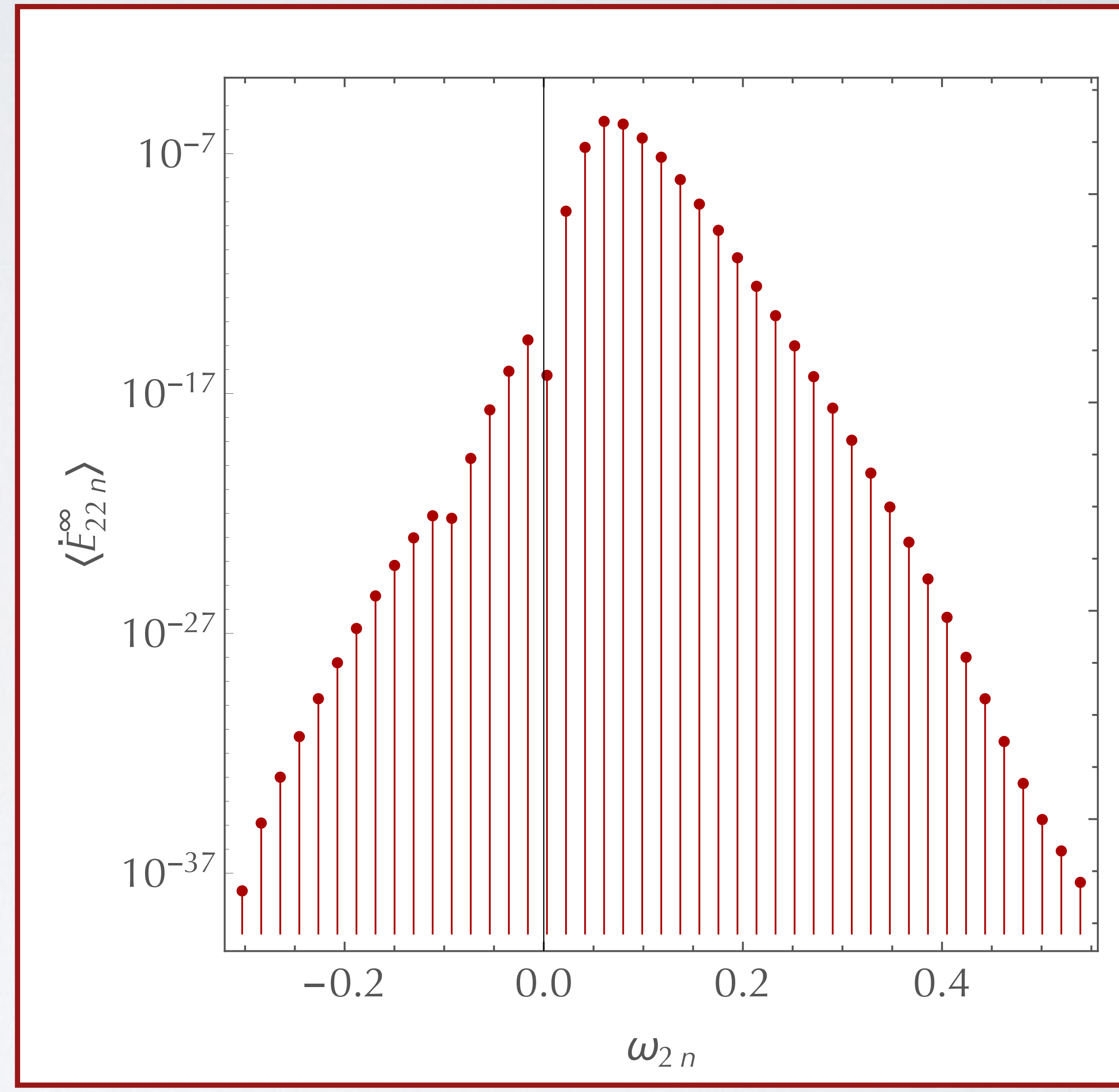

- Finite difference
- Pseudo-spectral

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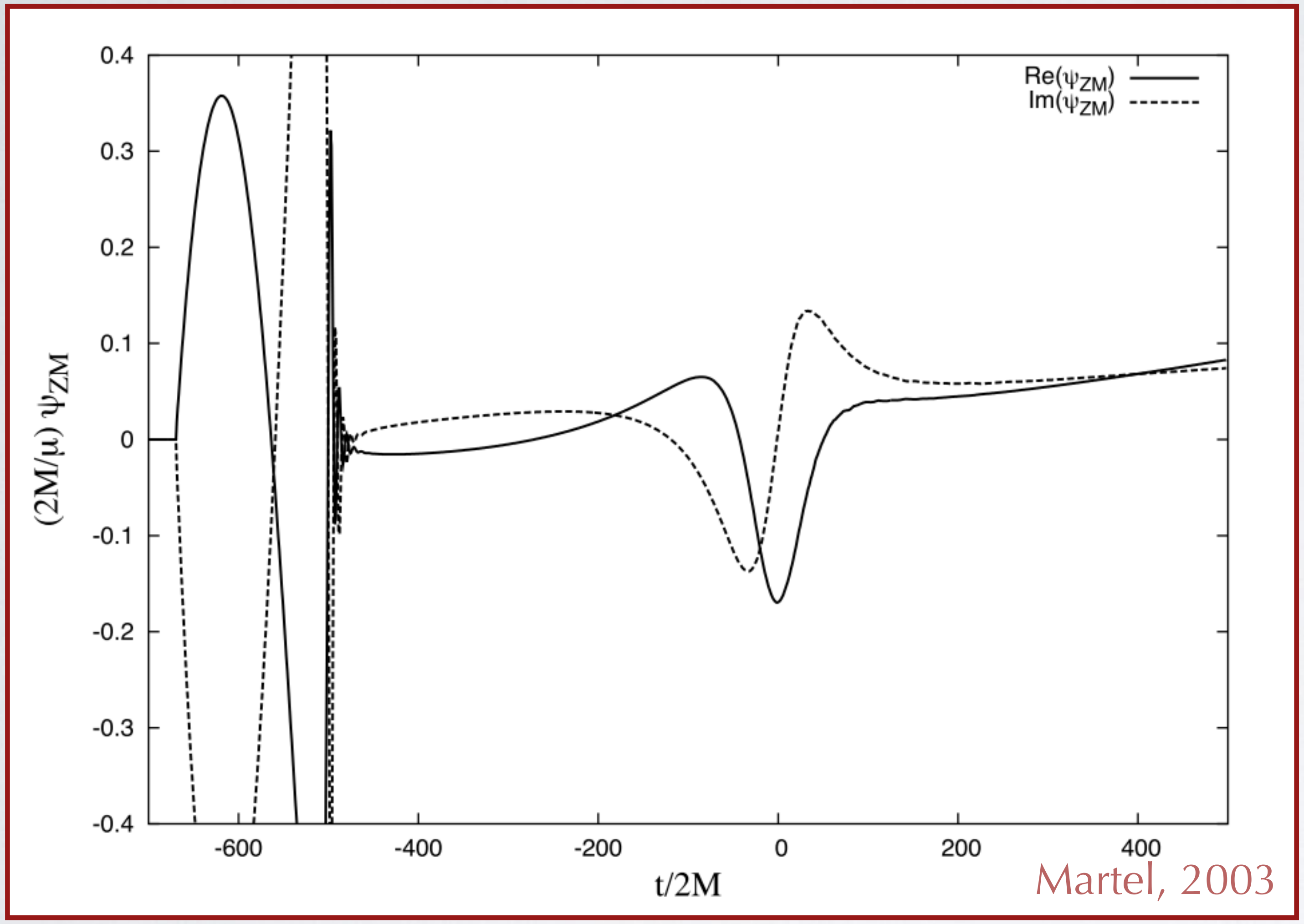
Time domain



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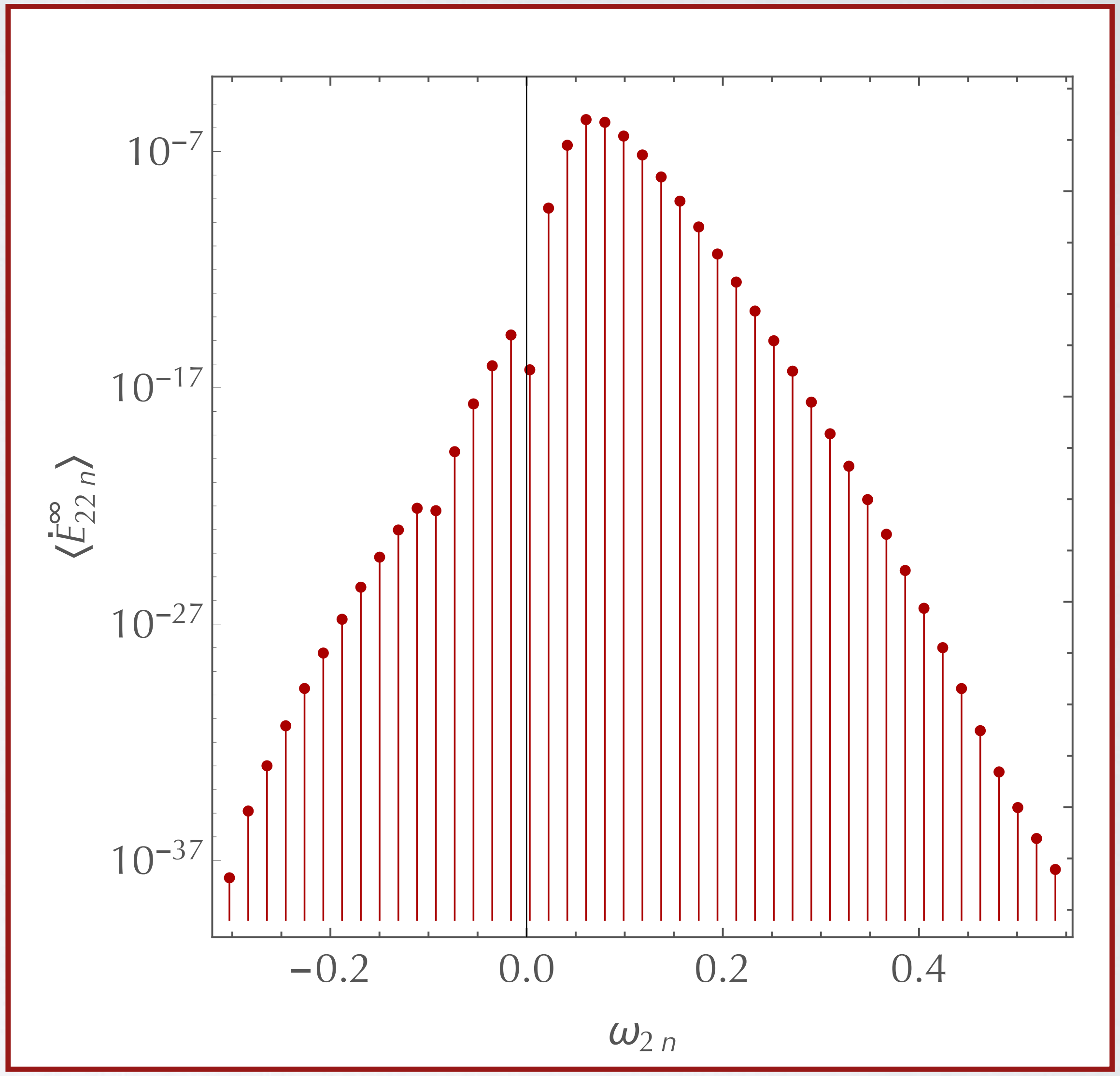

- Finite difference
- Pseudo-spectral
- Discontinuous Galerkin

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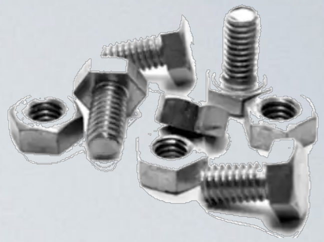

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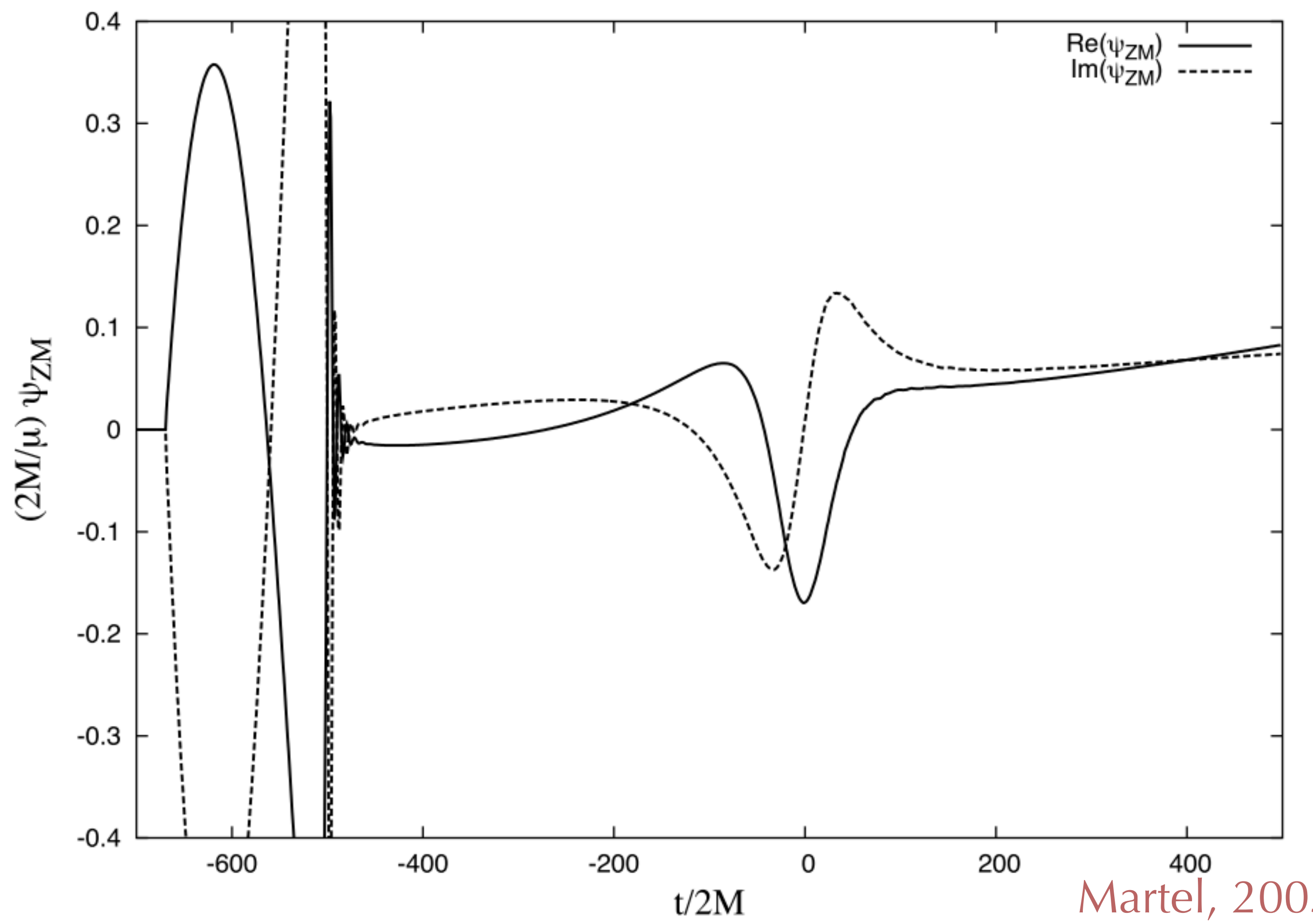

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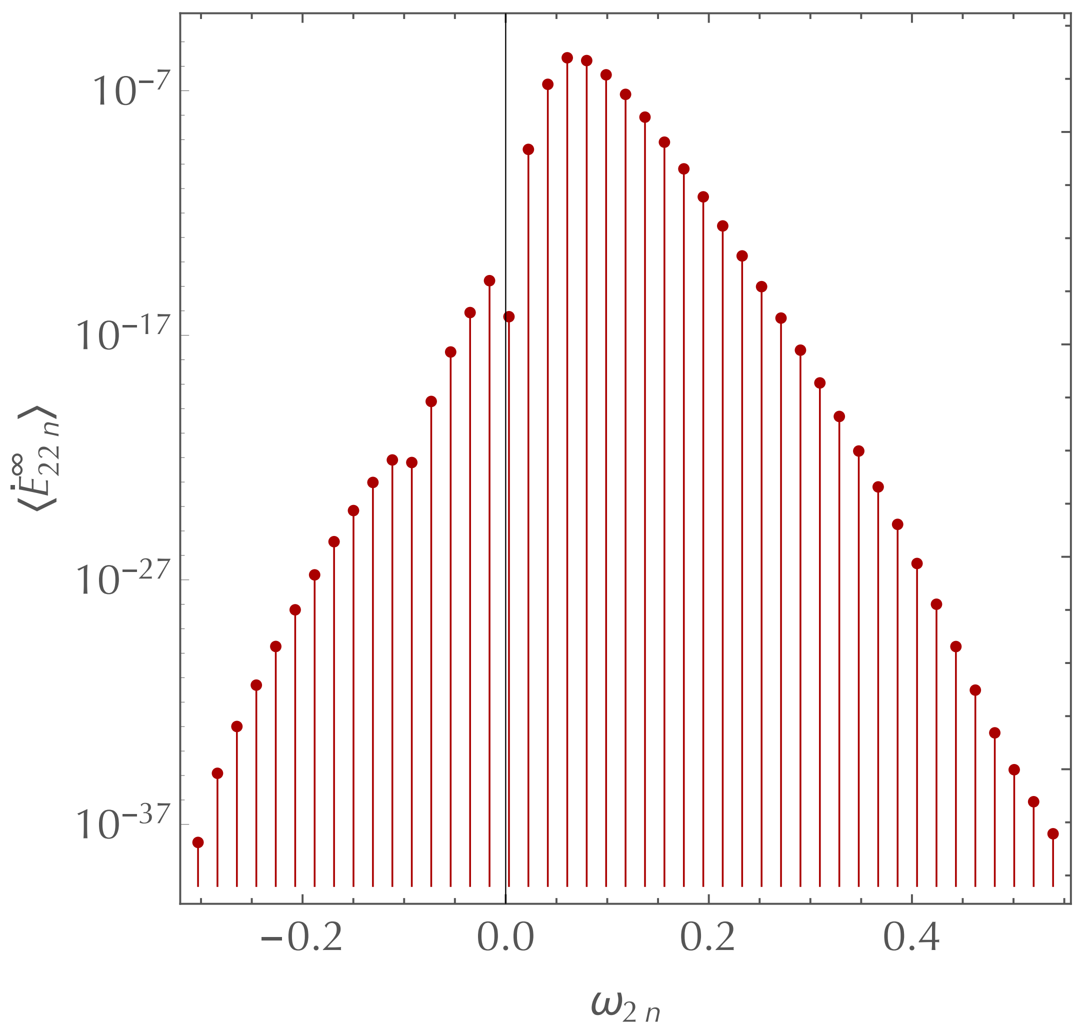

- Finite difference
- Pseudo-spectral
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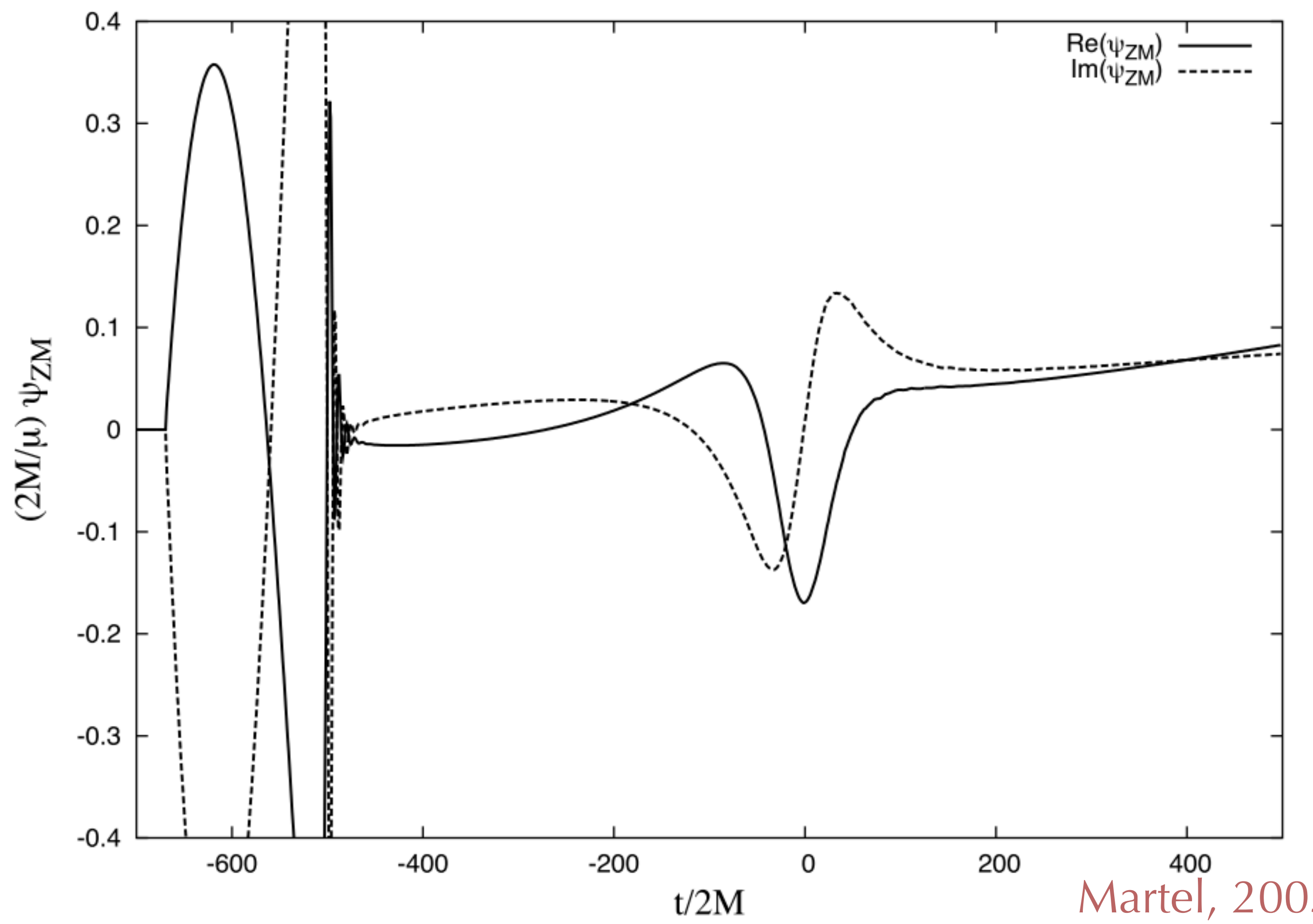
- 2 space + 1 time

There are many methods for solving the differential equations

Time domain



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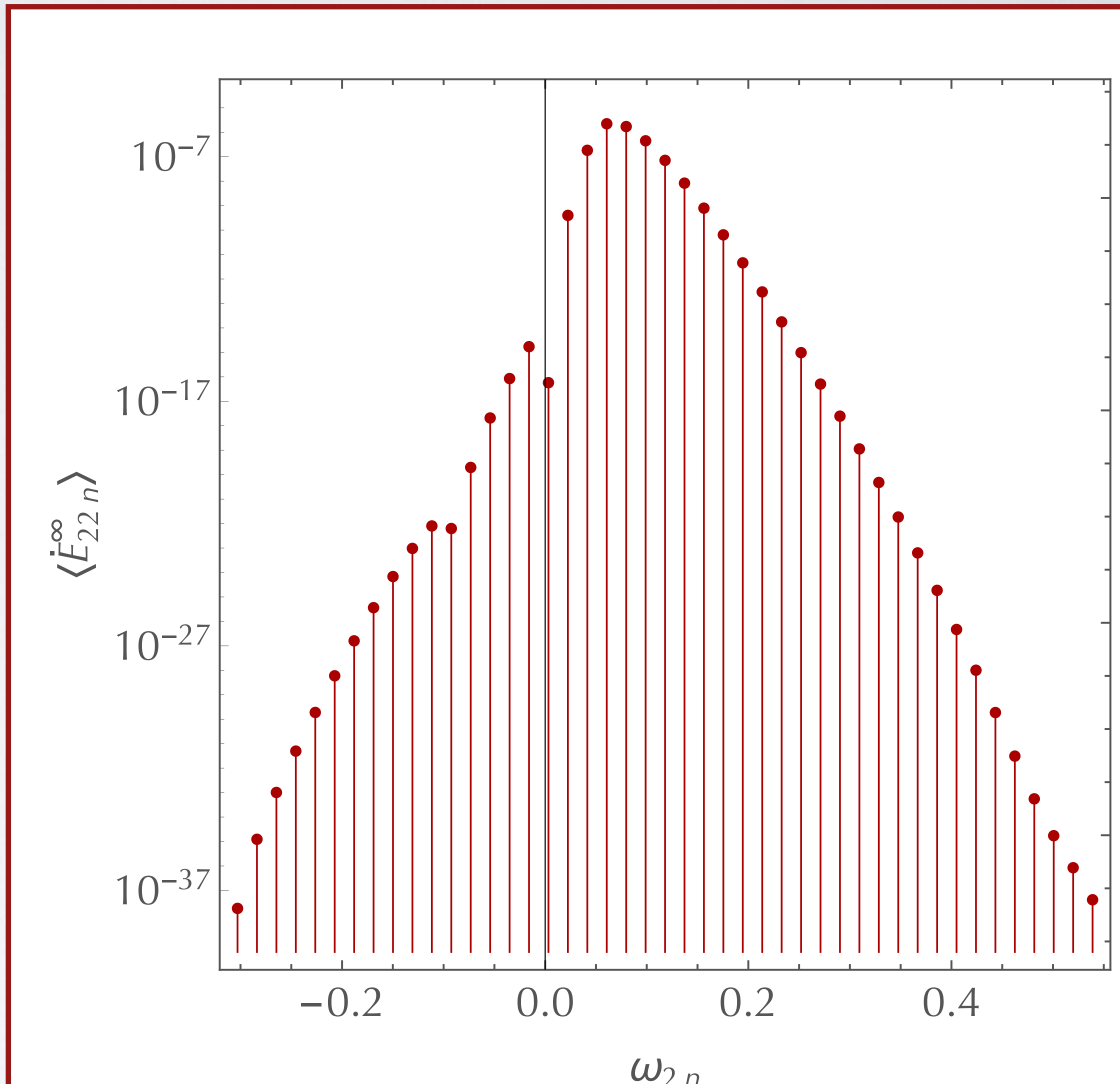

- Numerical integration

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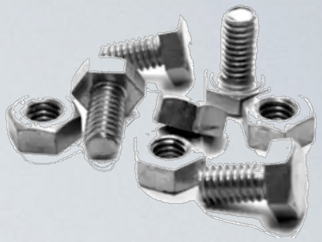

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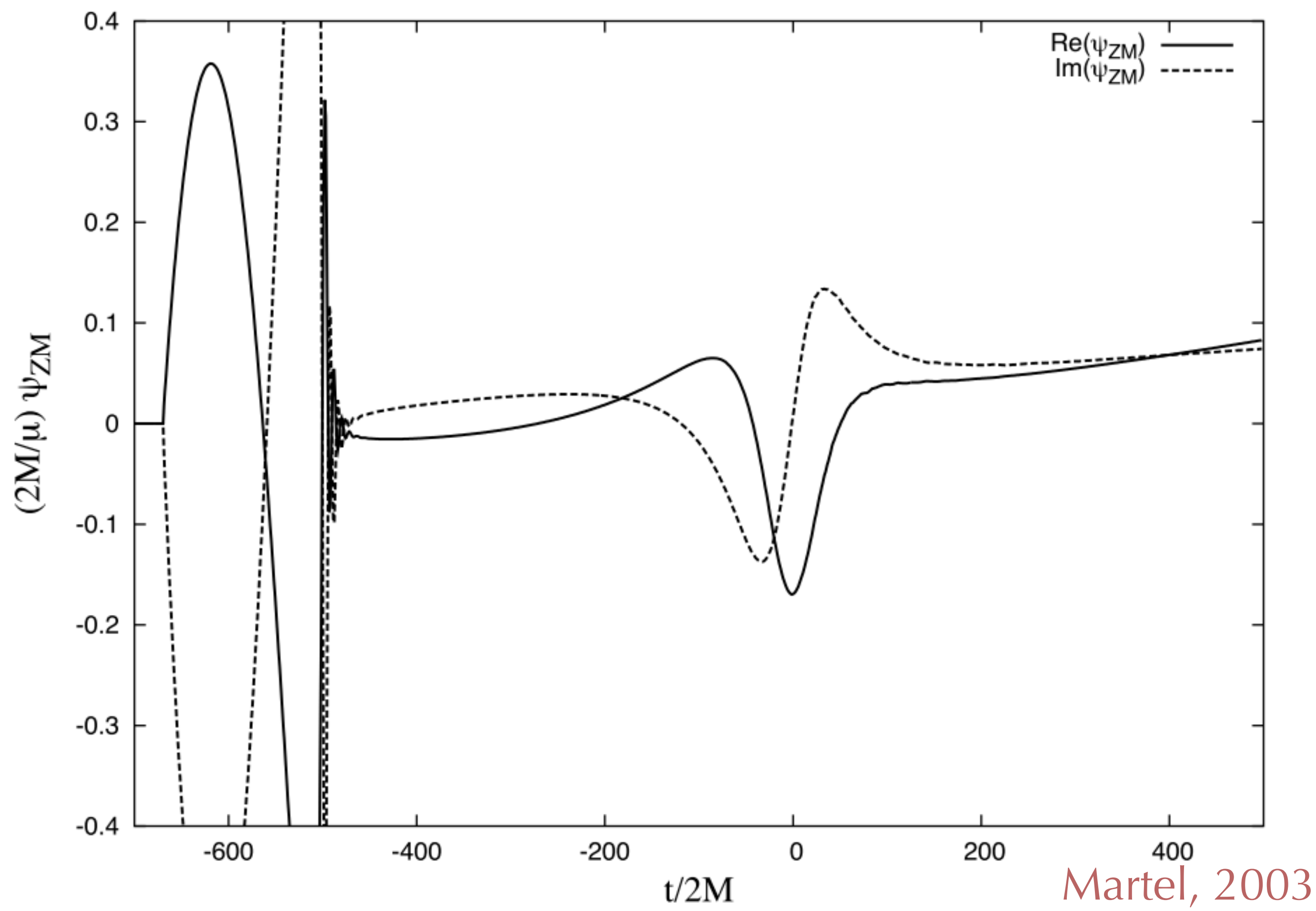

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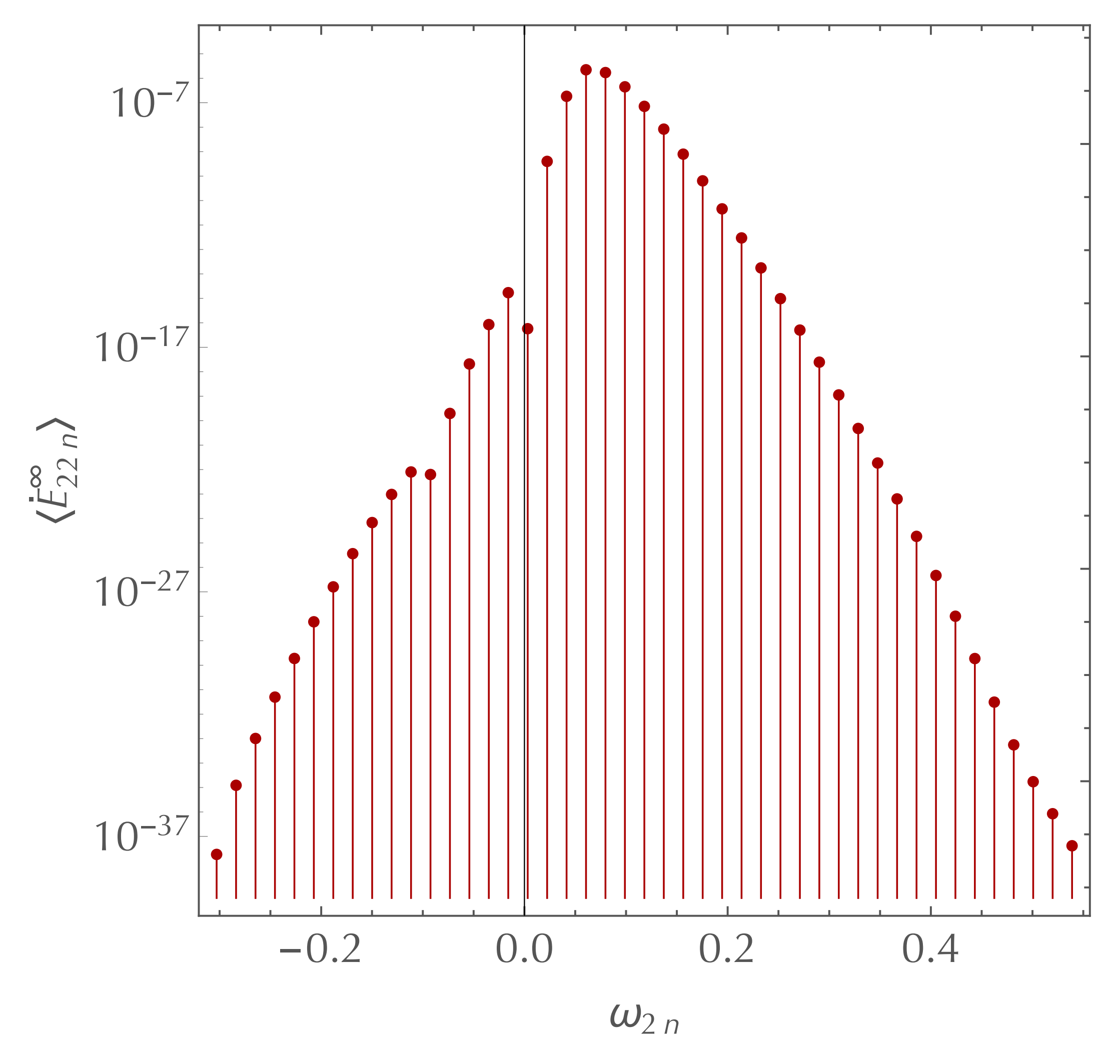

- Numerical integration
- Numeric MST

There are many methods for solving the differential equations

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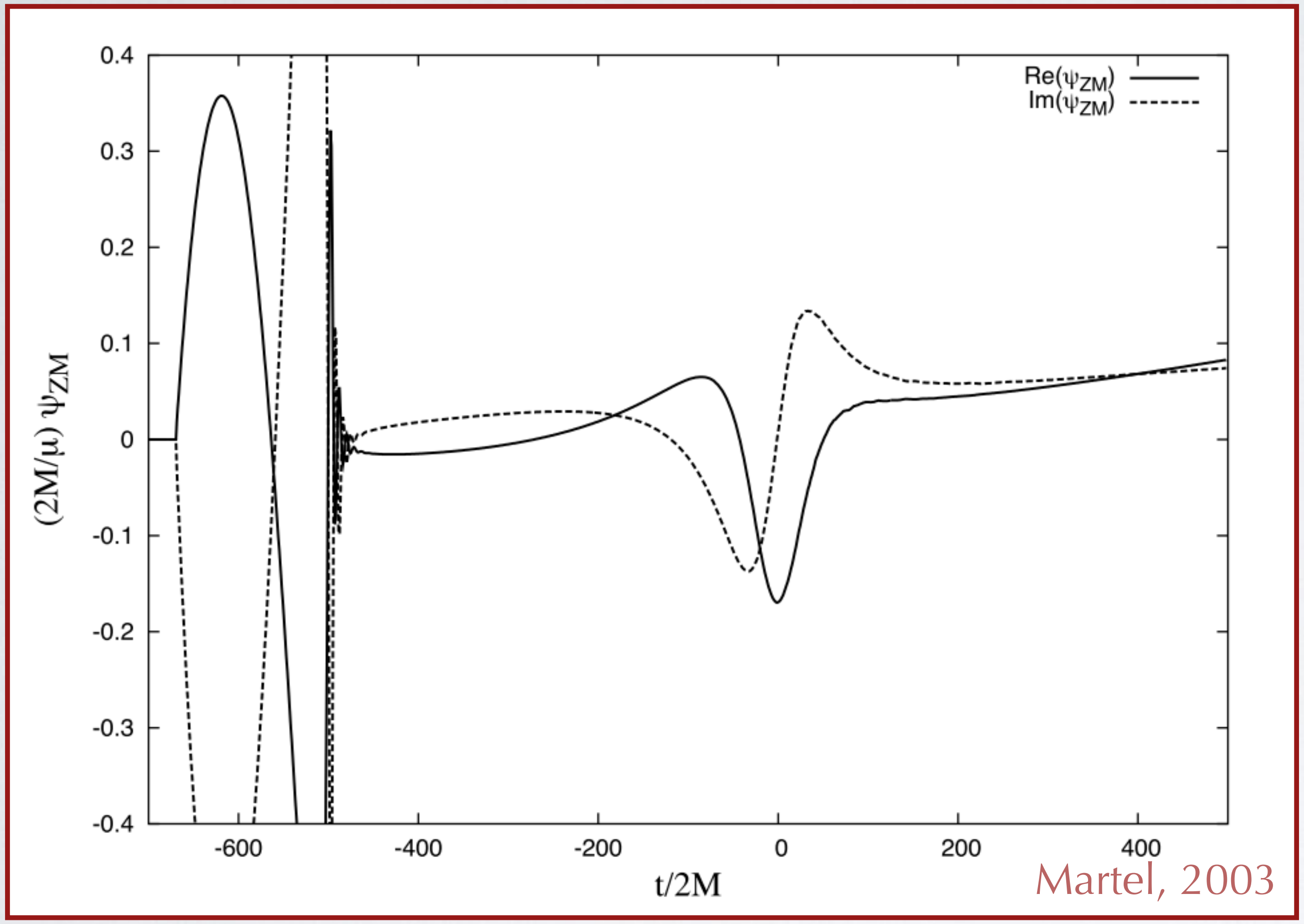

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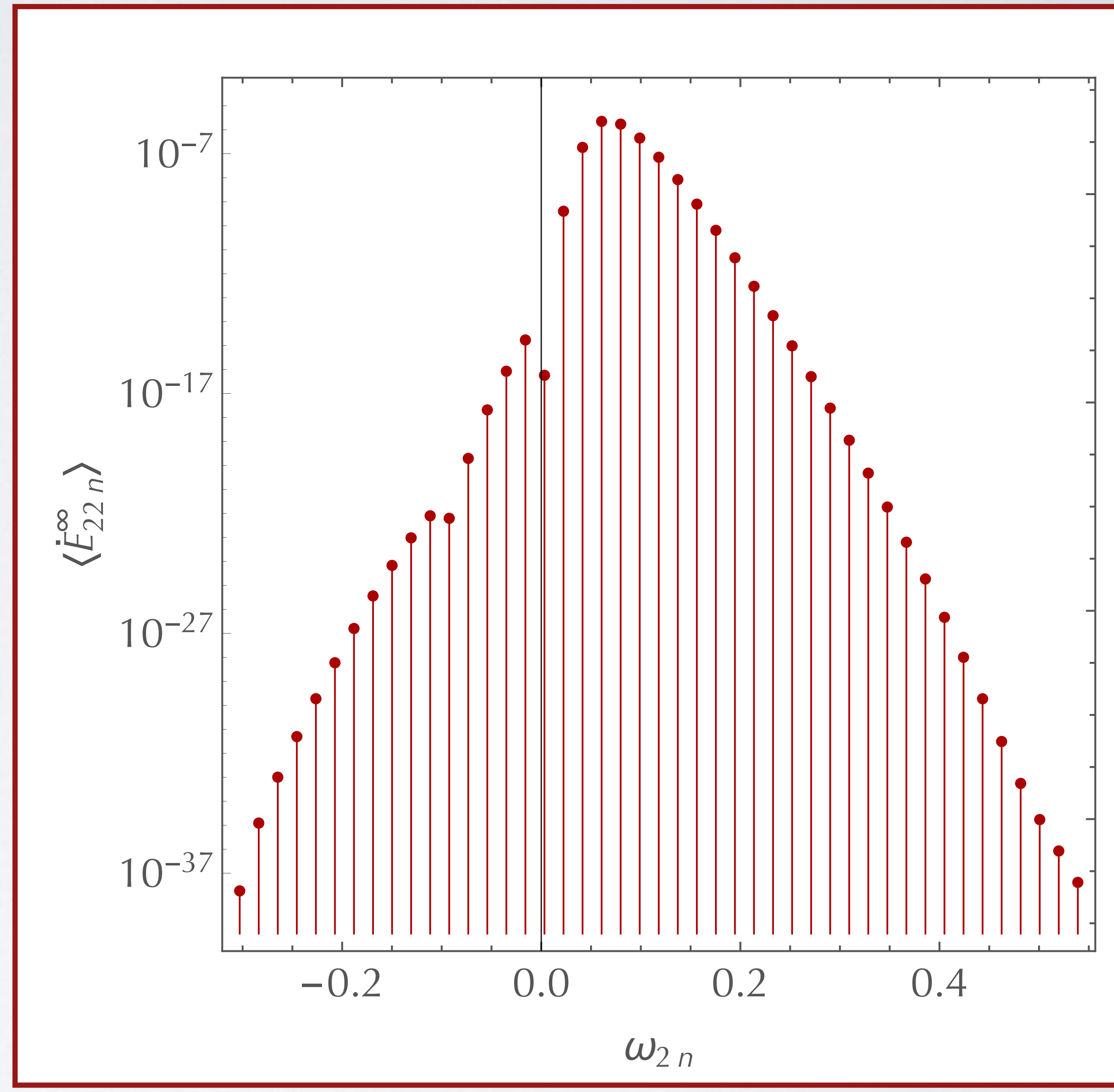

- Finite difference
- Pseudo-spectral
- Discontinuous Galerkin
- 2 space + 1 time

- Numerical integration
- Numeric MST
- Analytic MST + PN

Your choice of gauge will affect your result

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Regge-Wheeler

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- Not defined on Kerr

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Lorenz

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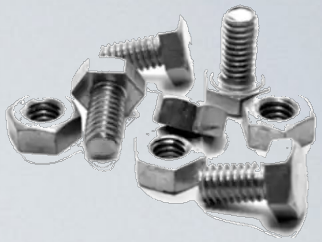

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- Reconstruction is a bit messy
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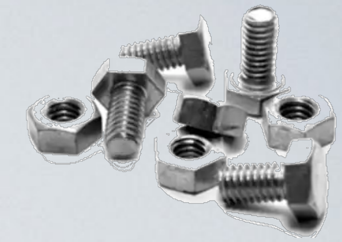

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Jonathan Thompson - Wednesday

Yesterday

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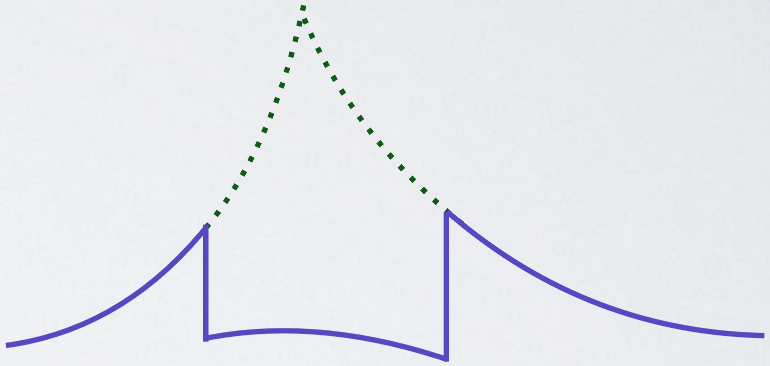
Regularization



Practical considerations

Gauge invariants

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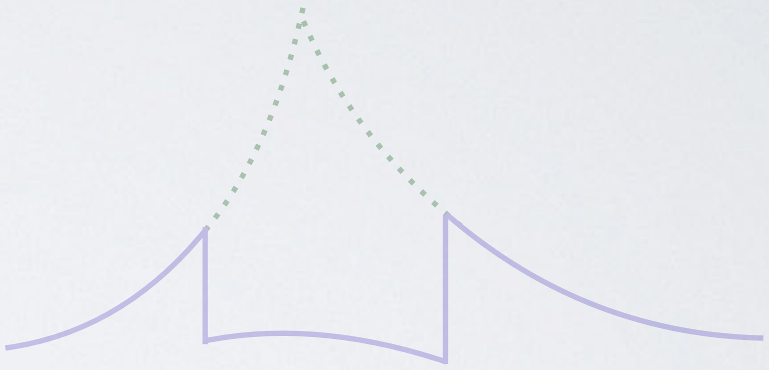
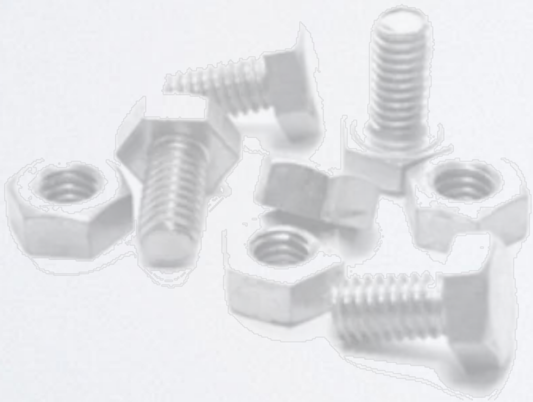
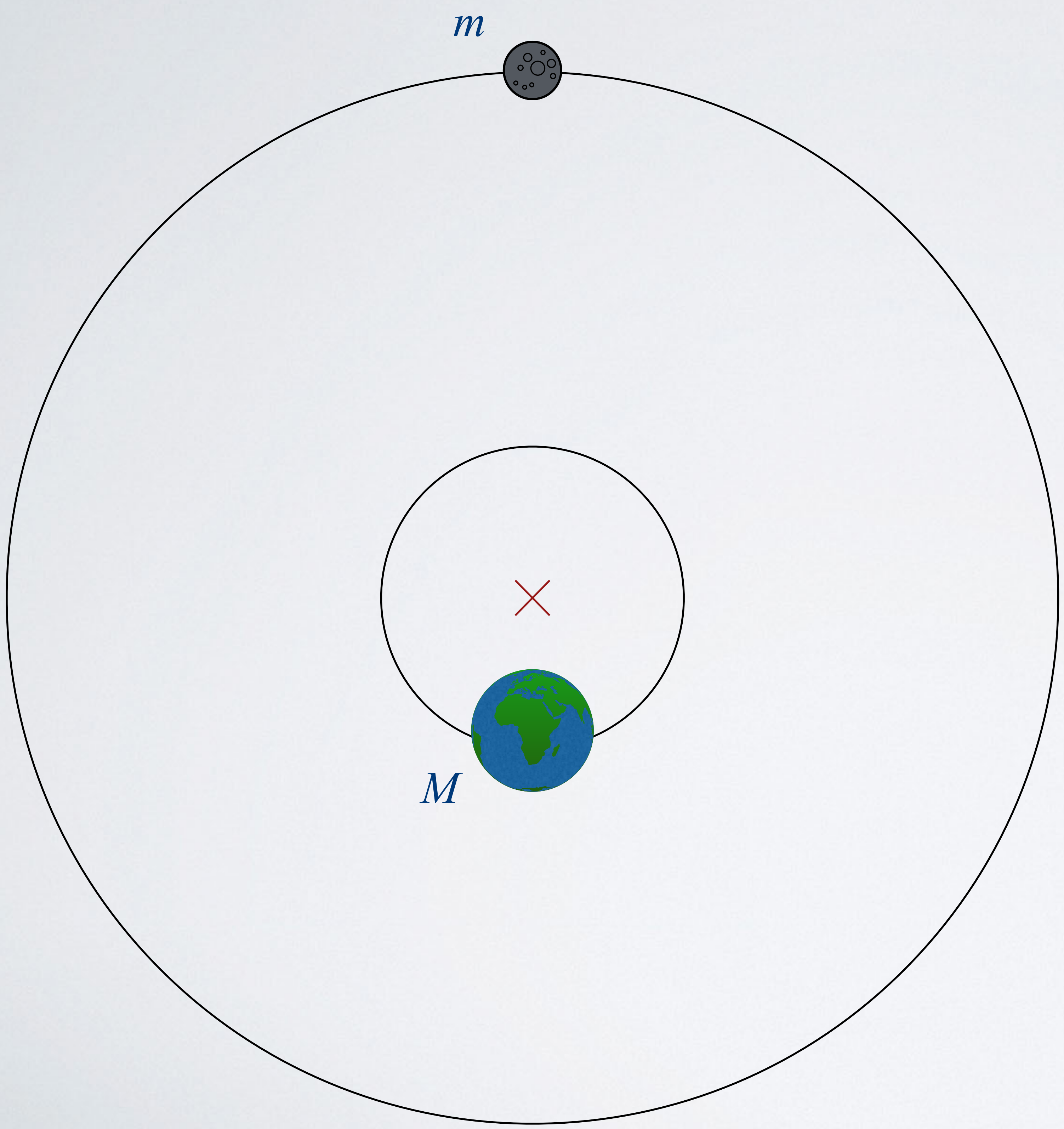


Practical considerations



Gauge invariants

Here's another way to irritate undergrads



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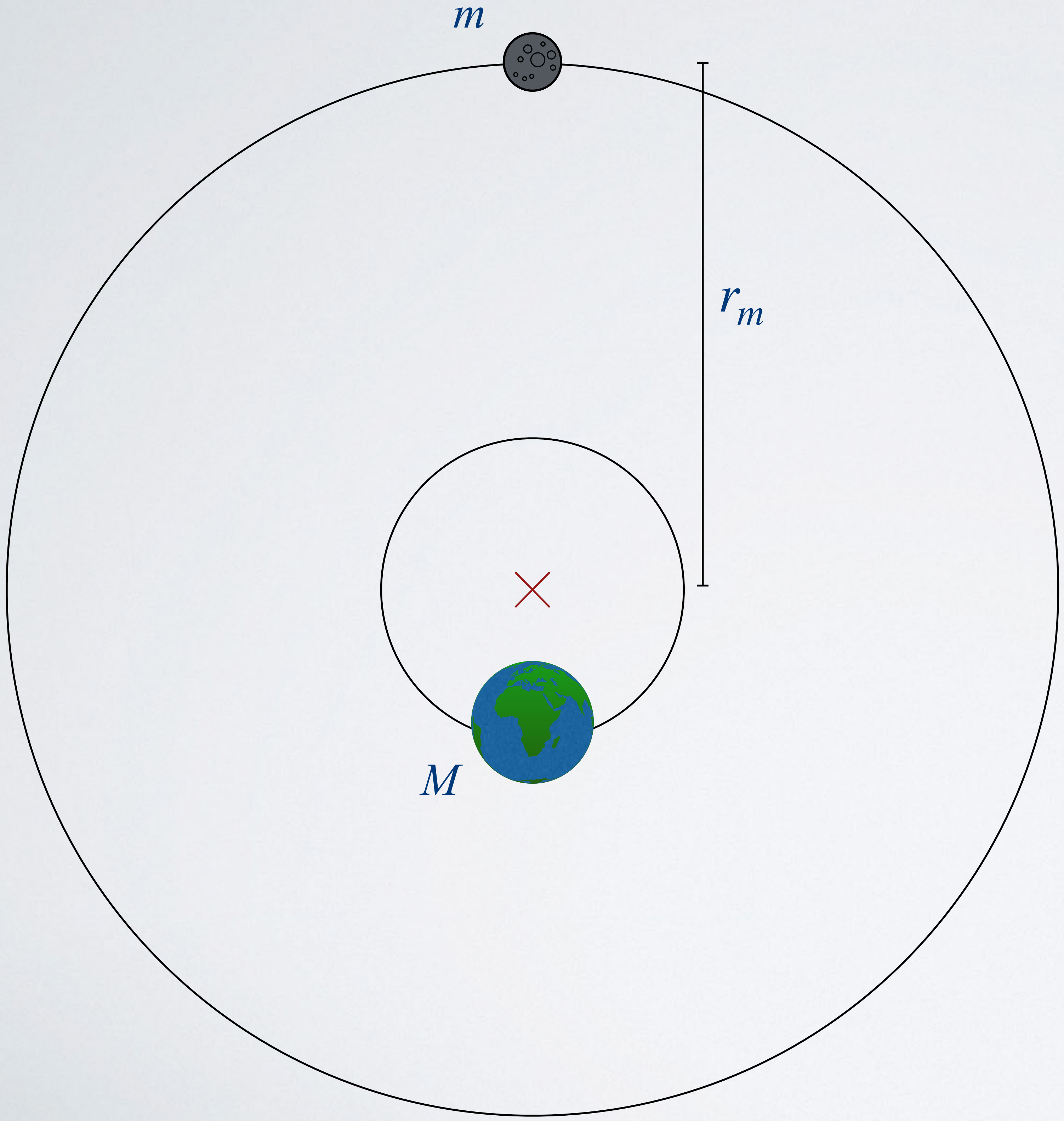

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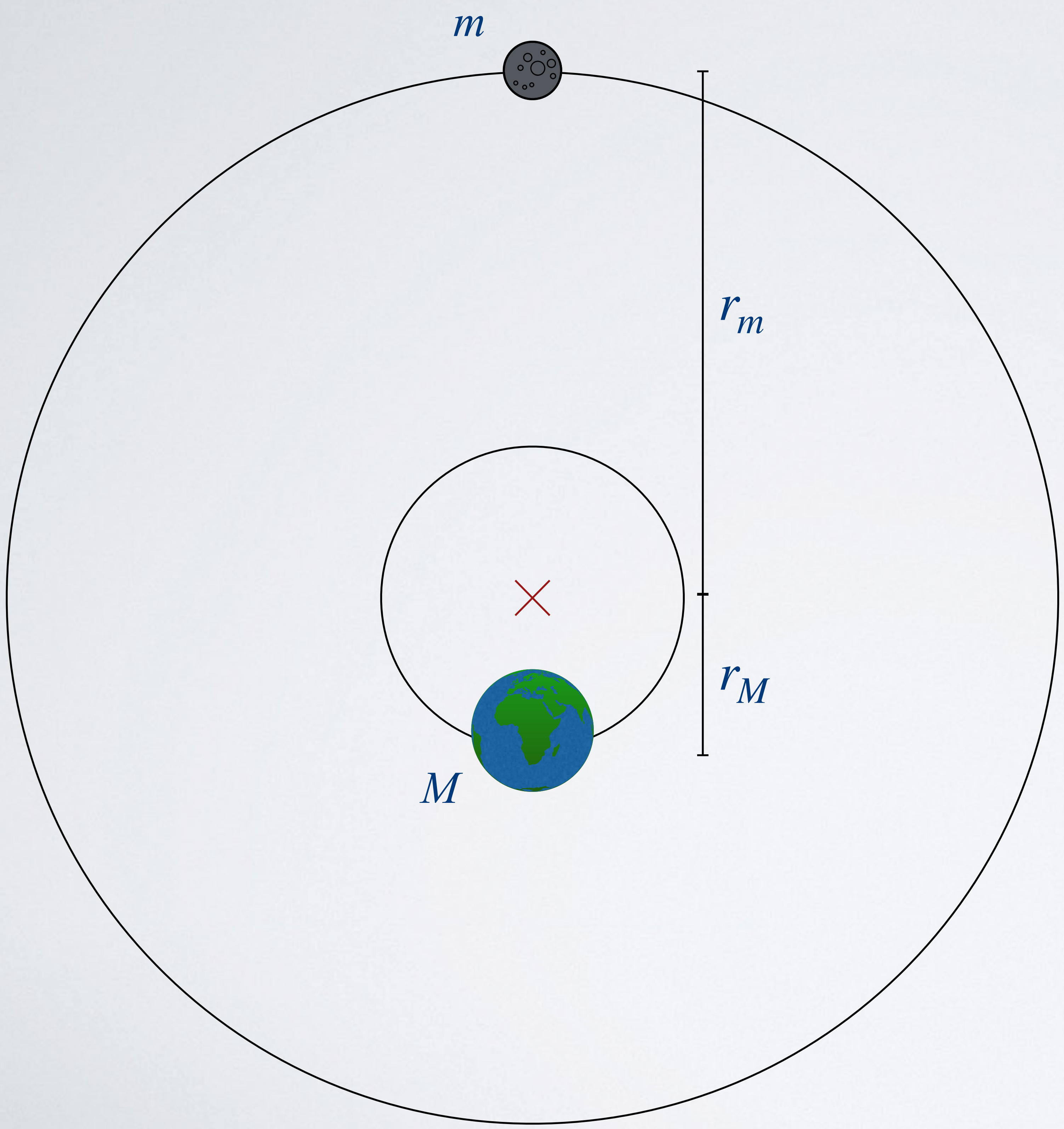

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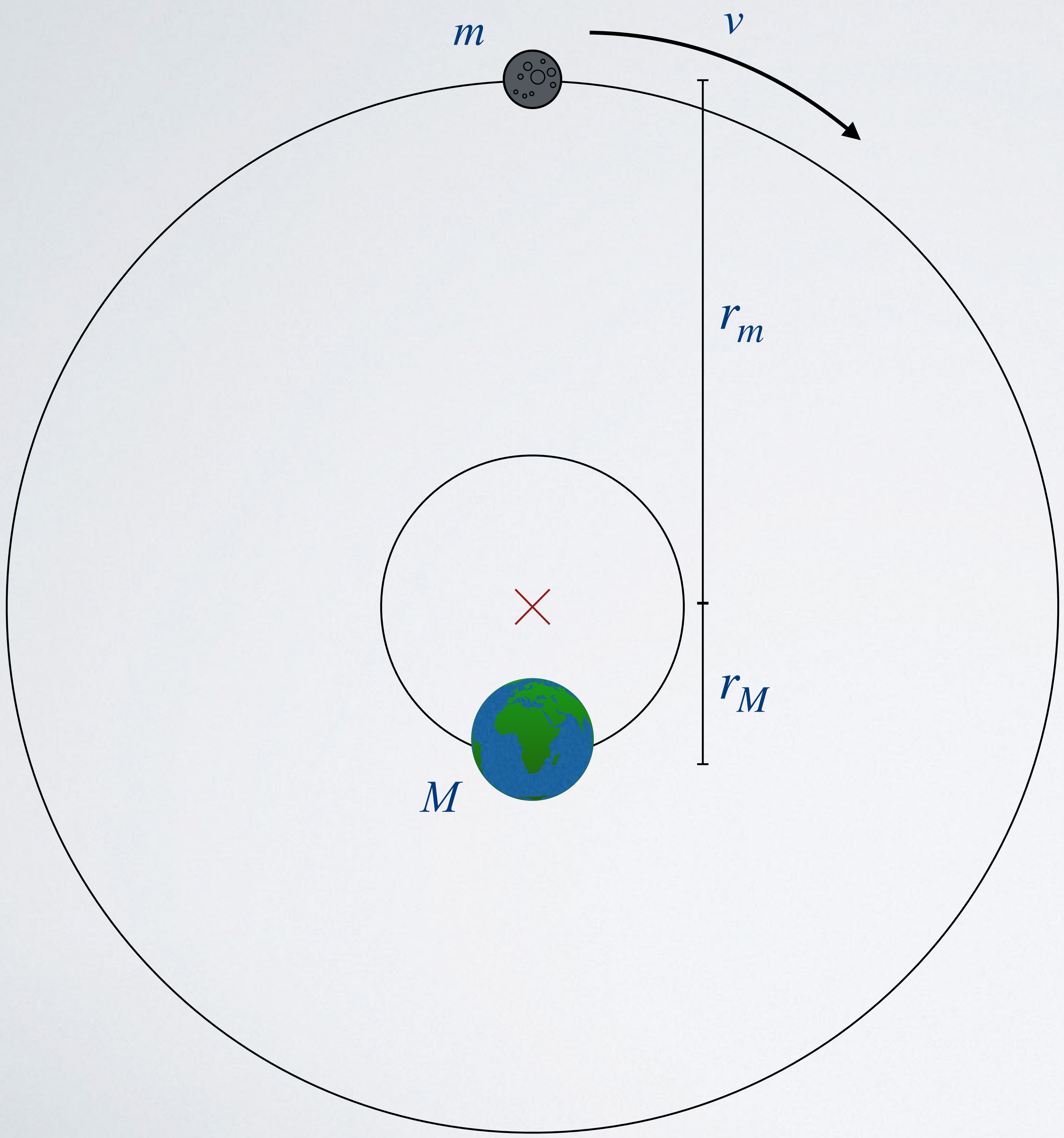

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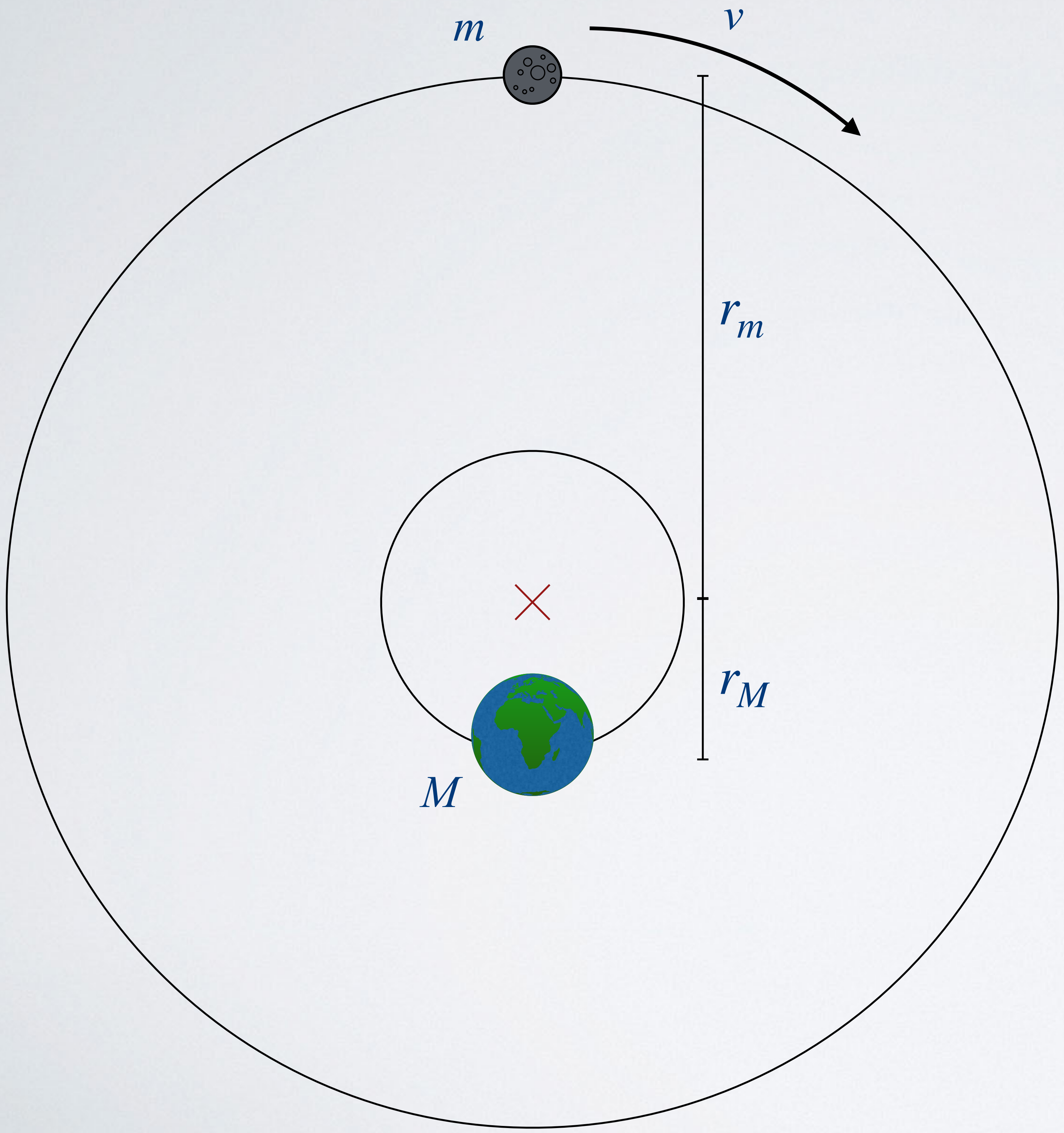

Steve Detweiler asks: What is the speed of the moon in the limit that $m/M \ll 1$?

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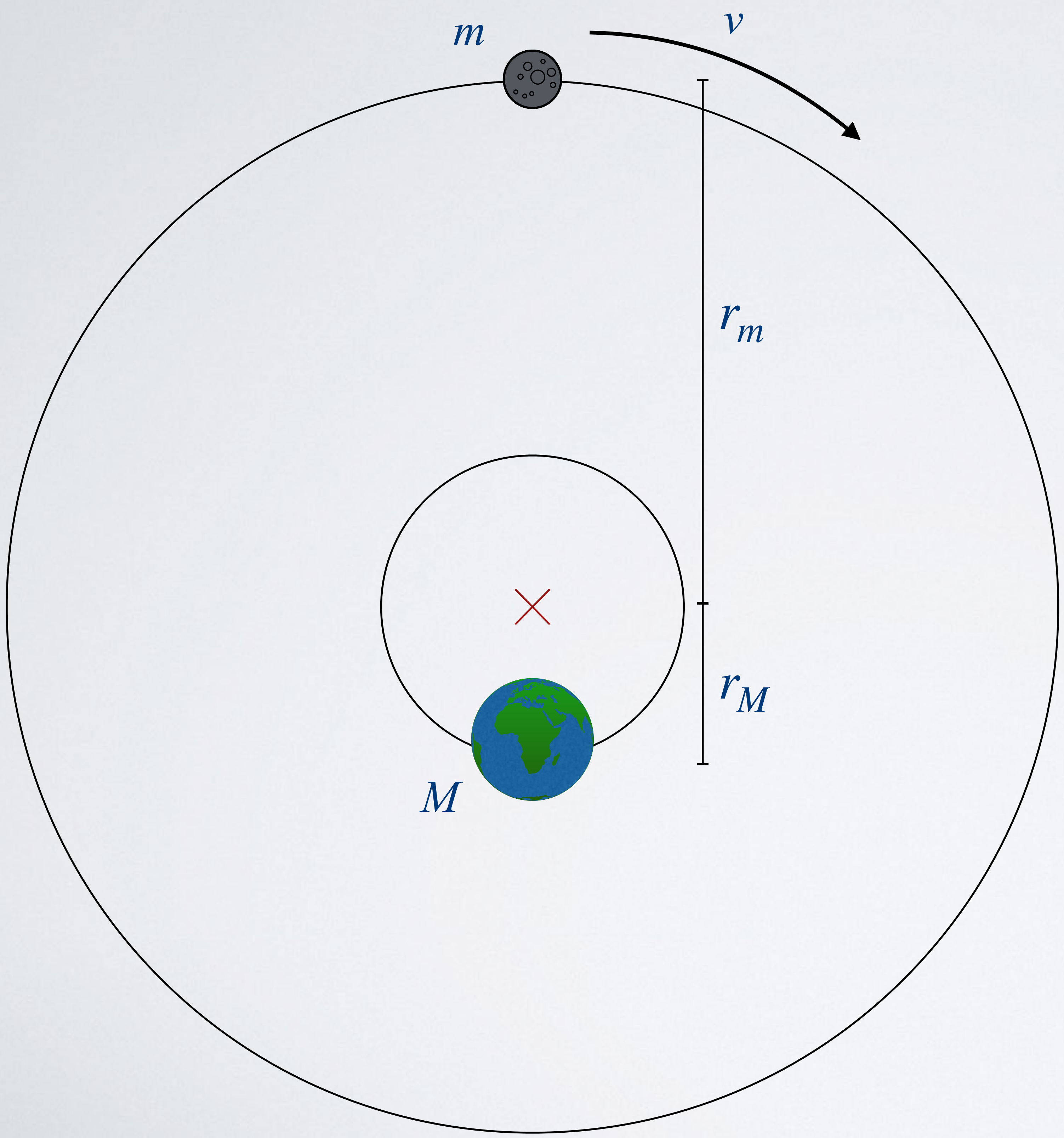

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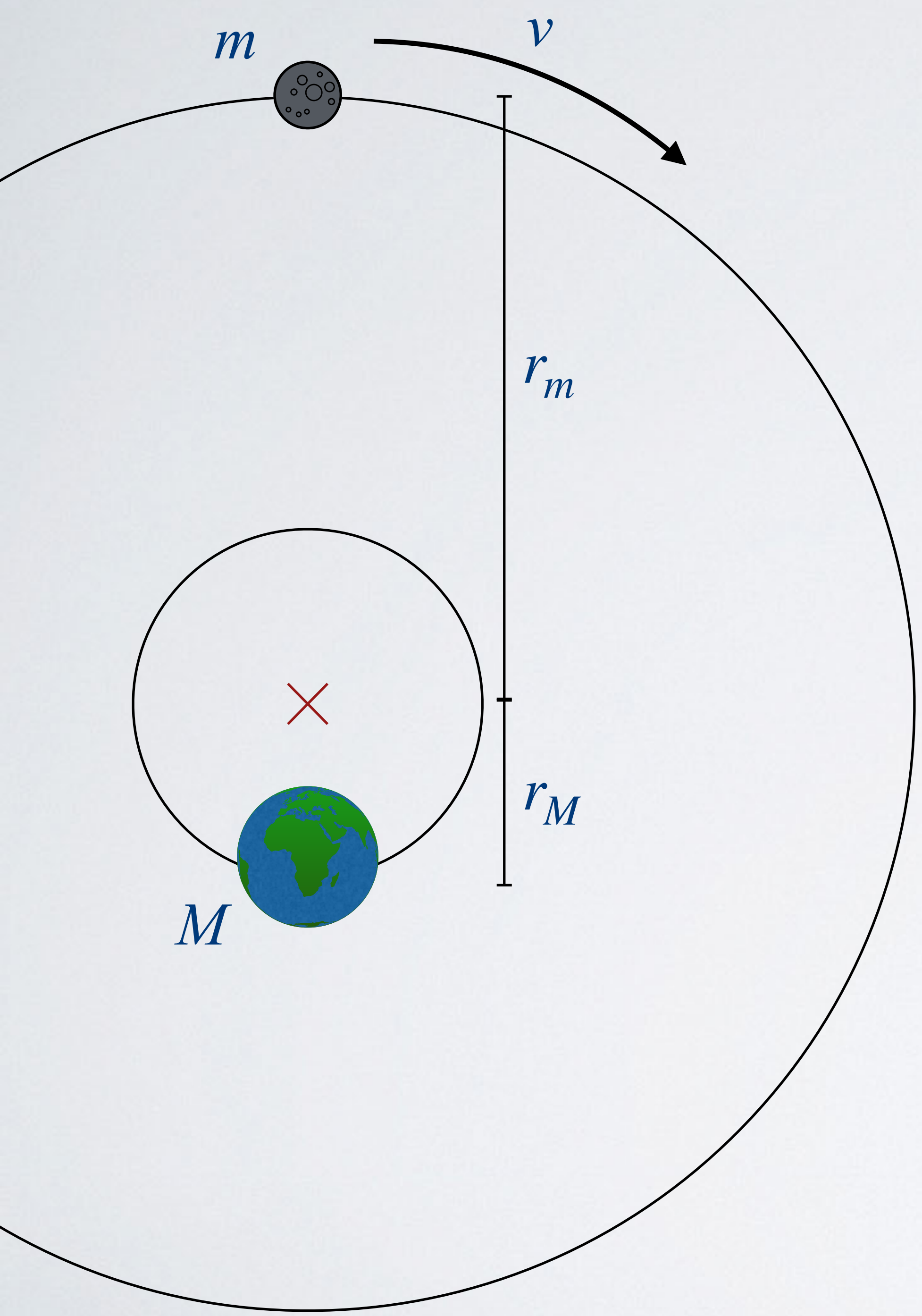
$$\frac{v^2}{r_m} = \frac{GM}{(r_m + r_M)^2}$$

Rewrite the speed as an expansion in the mass-ratio



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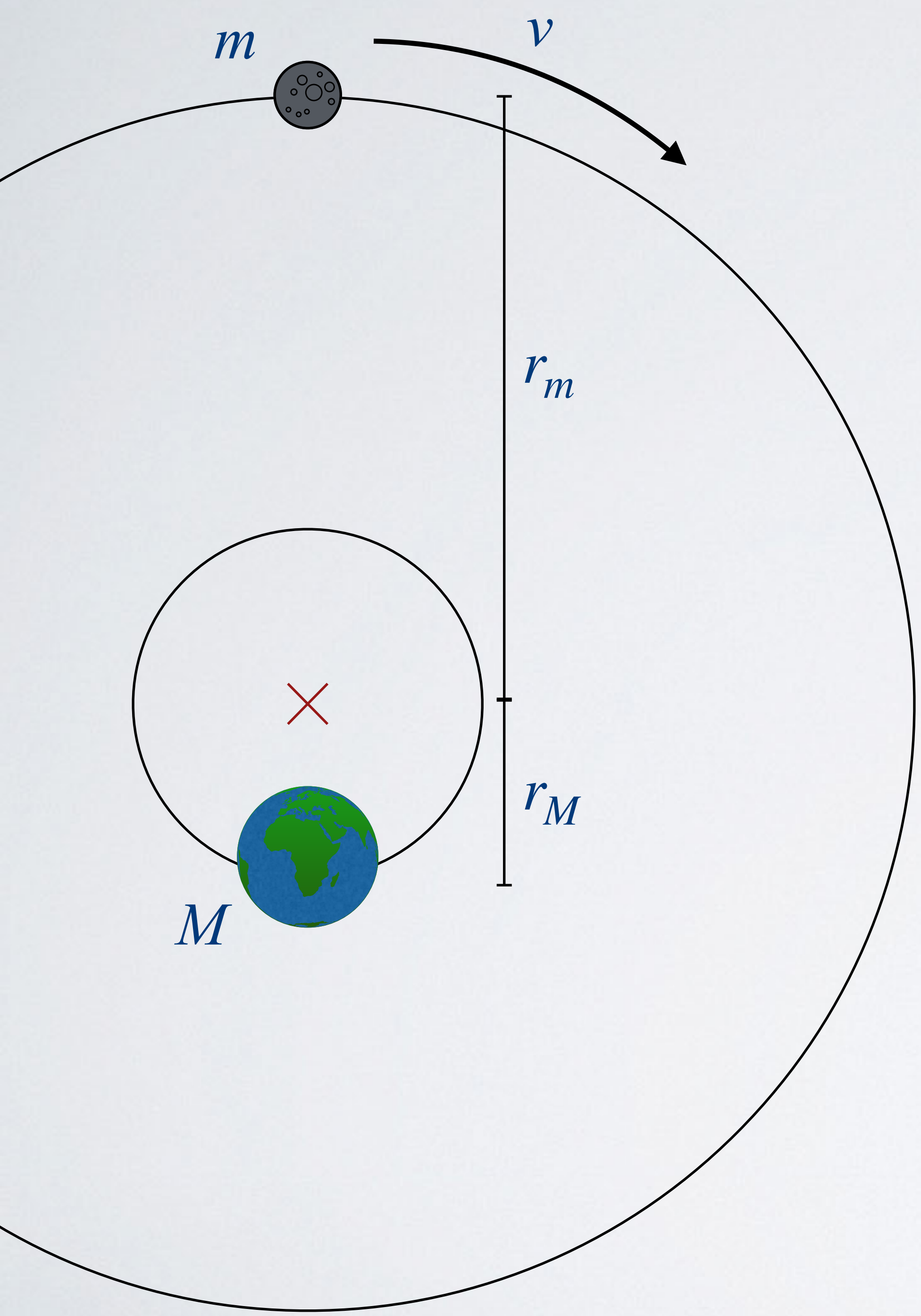

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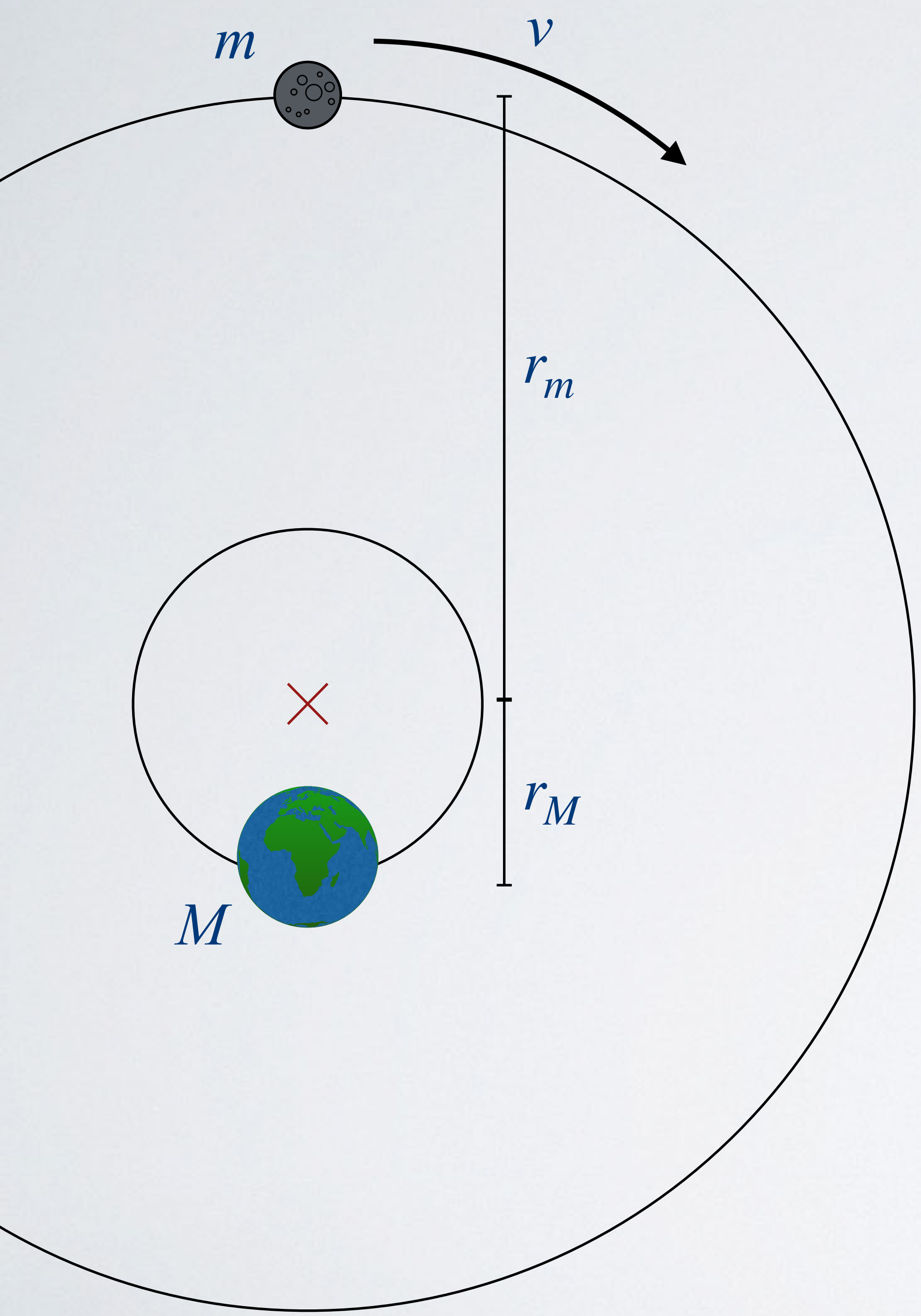

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$$v^2 = \frac{GM}{r_m}(1 - 2m/M + \cdots)$$

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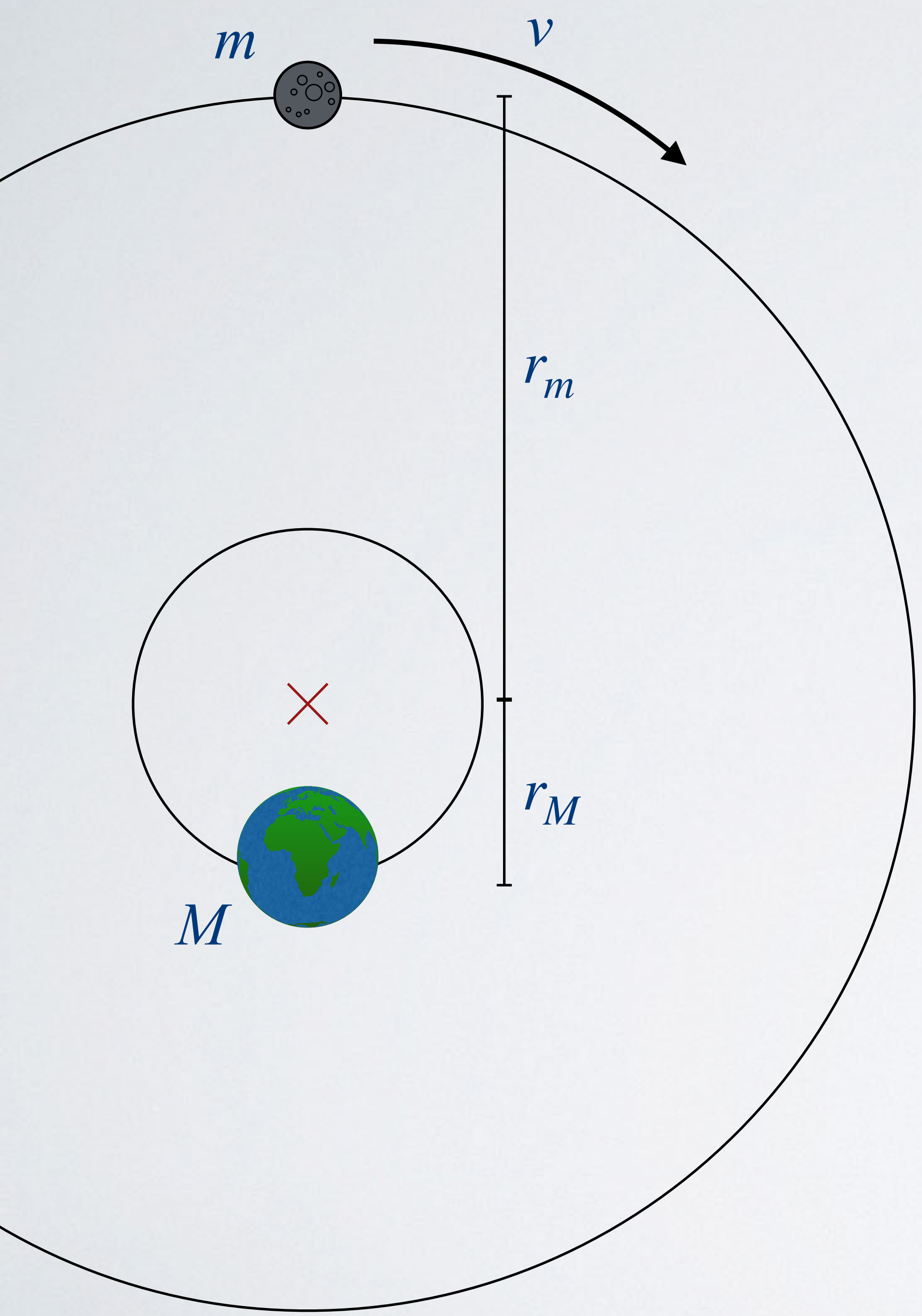

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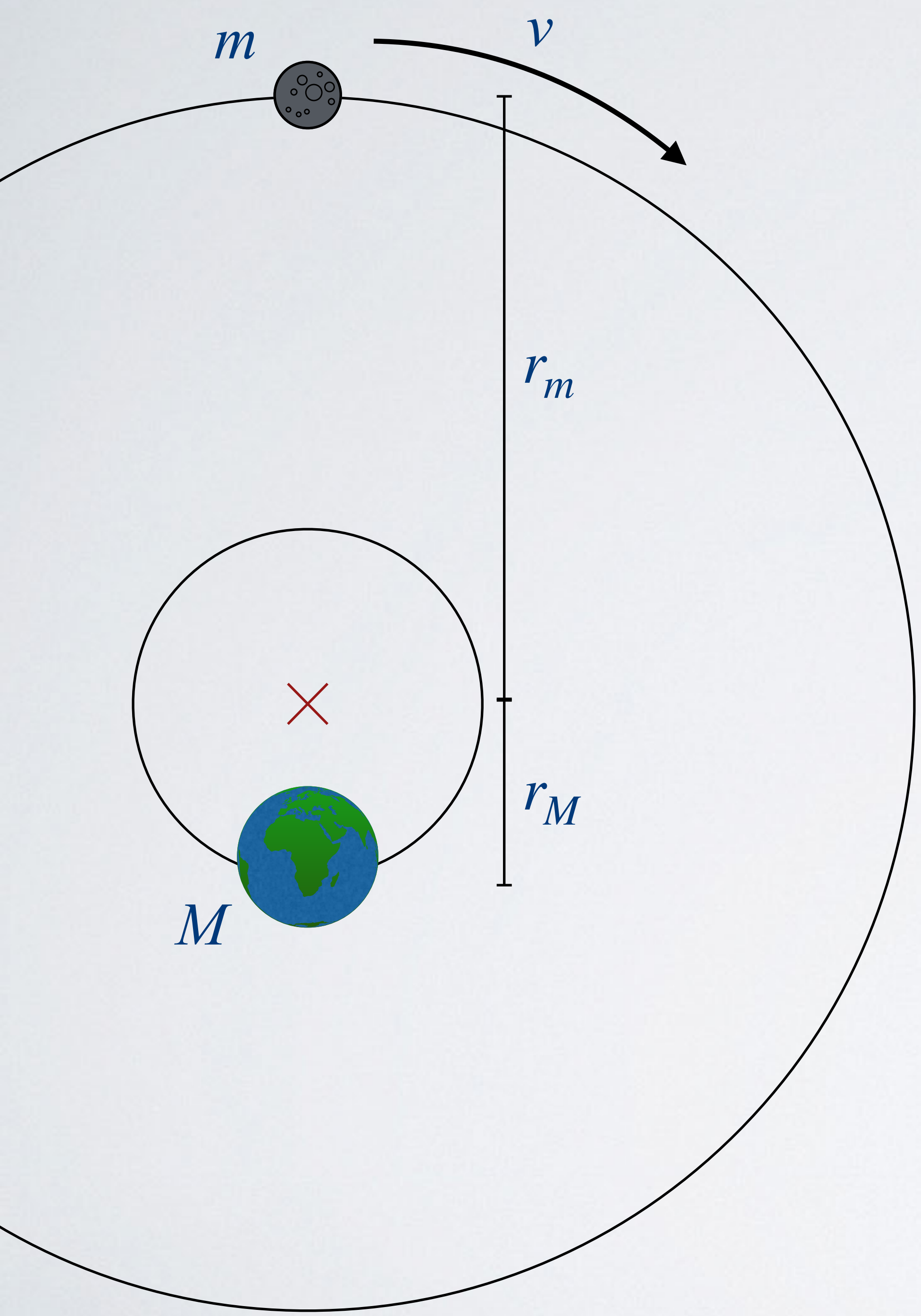
We can write a *different* expansion in the same mass-ratio



$$\frac{v^2}{r_m} = \frac{GM}{R^2}$$

$$R \equiv r_m + r_M$$

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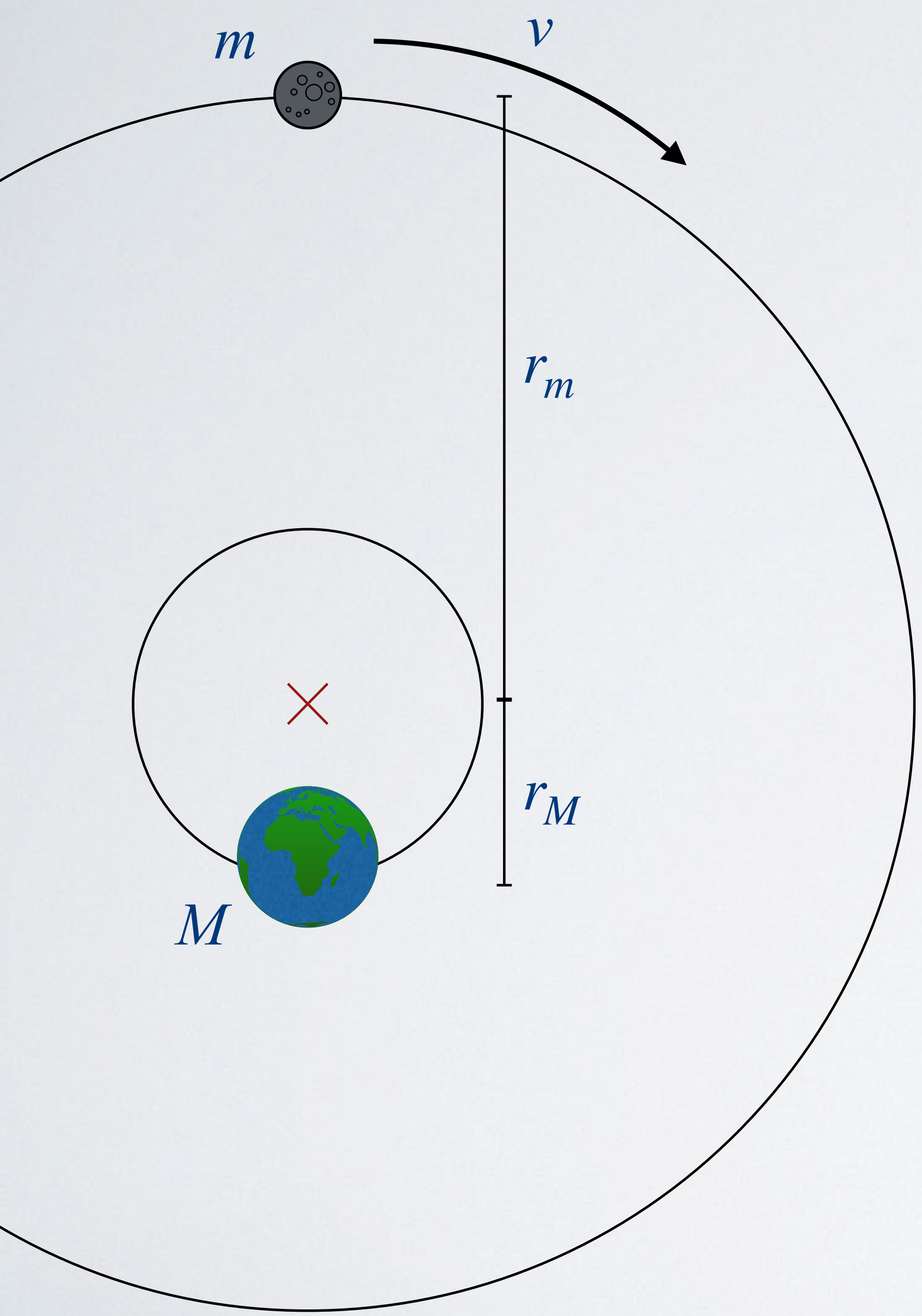

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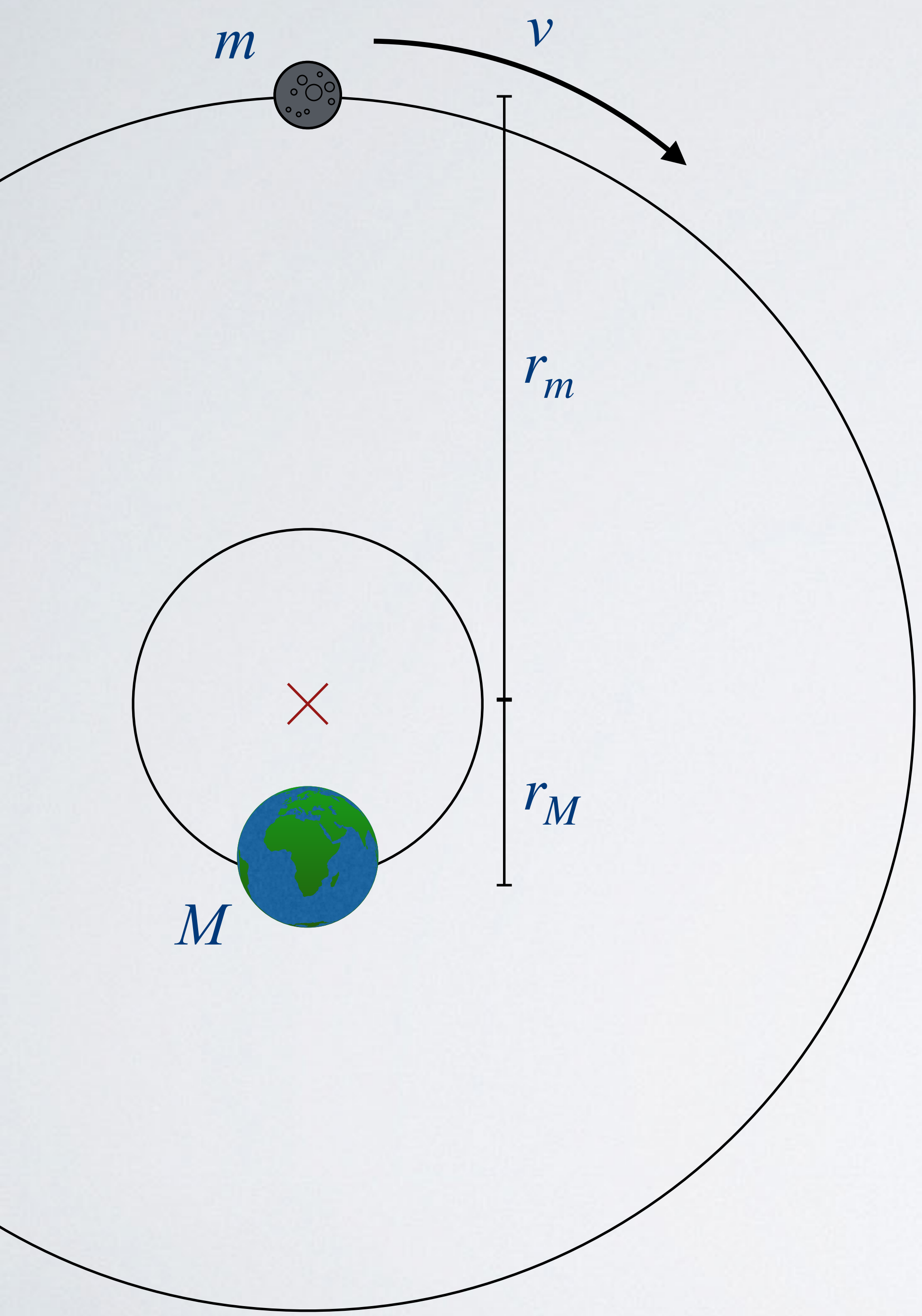

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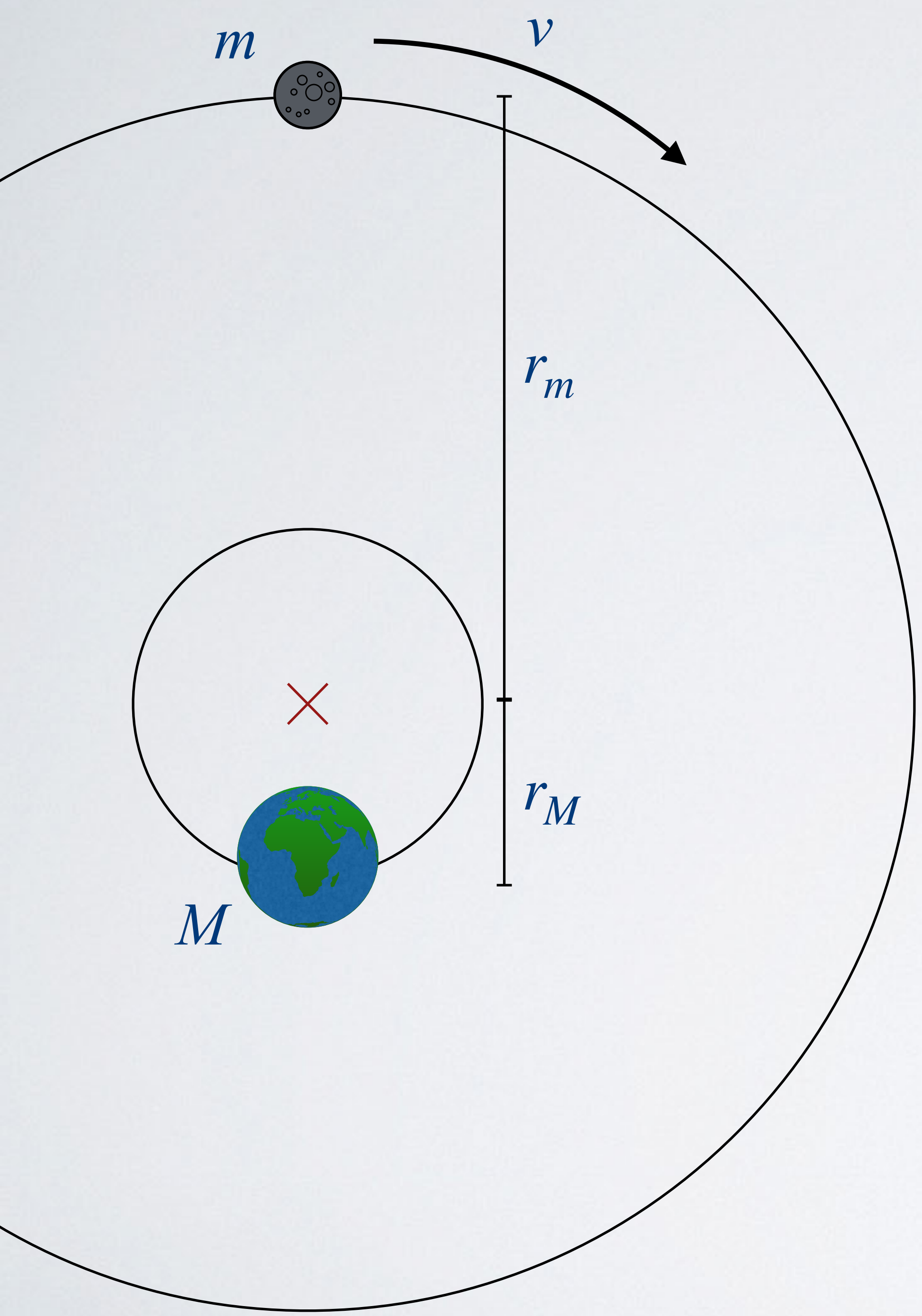
We can write a *different* expansion in the same mass-ratio

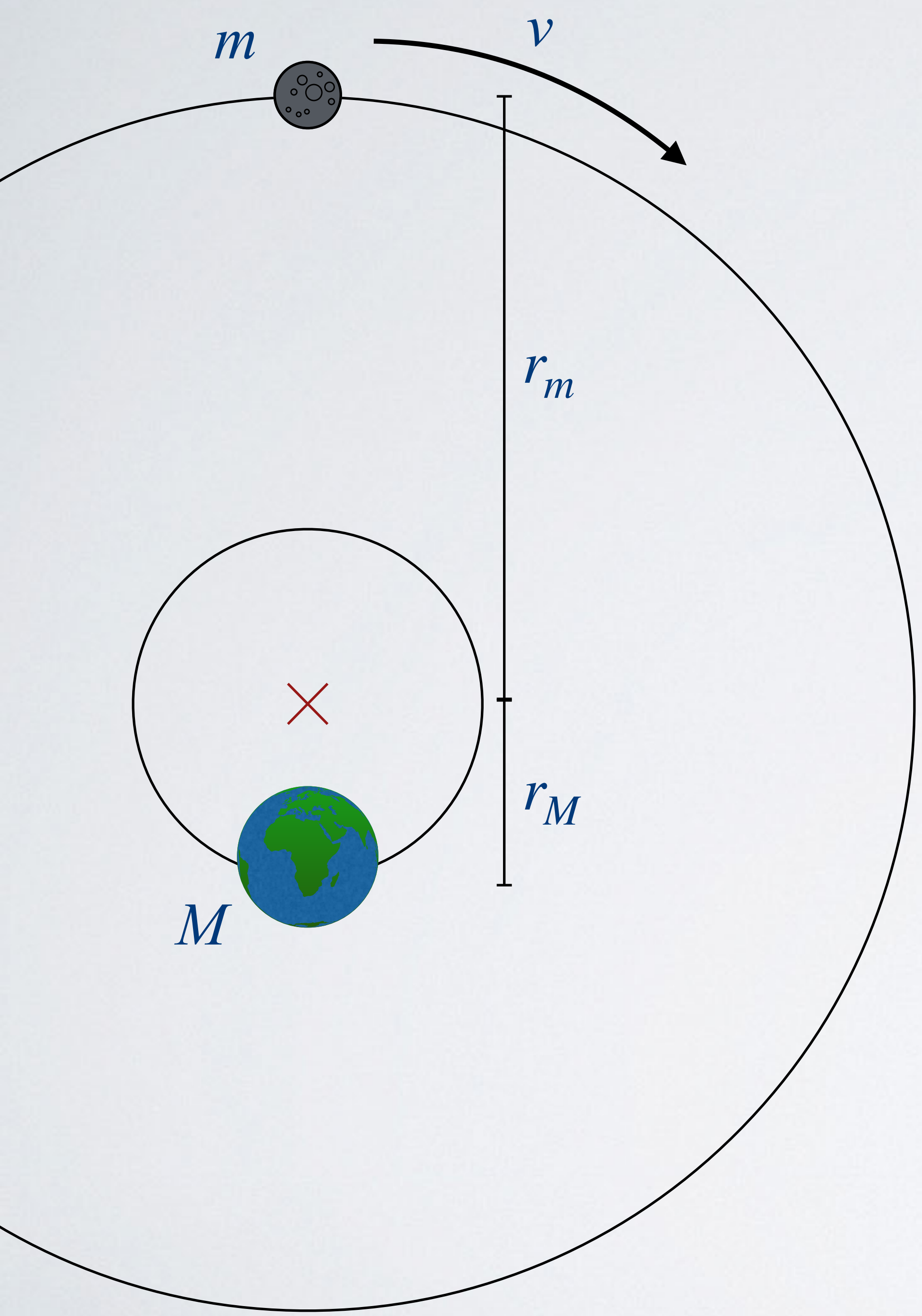


$$\frac{v^2}{r_m} = \frac{GM}{R^2} \qquad R \equiv r_m + r_M$$

$$v^2 = \frac{GM}{R(1 + m/M)}$$

$$v^2 = \frac{GM}{R}(1 \boxed{- m/M} + \cdots)$$

We have implicitly held different quantities constant, giving seemingly contradictory results



$$v^2 = \frac{GM}{r_m}(1 \boxed{- 2m/M} + \cdots)$$

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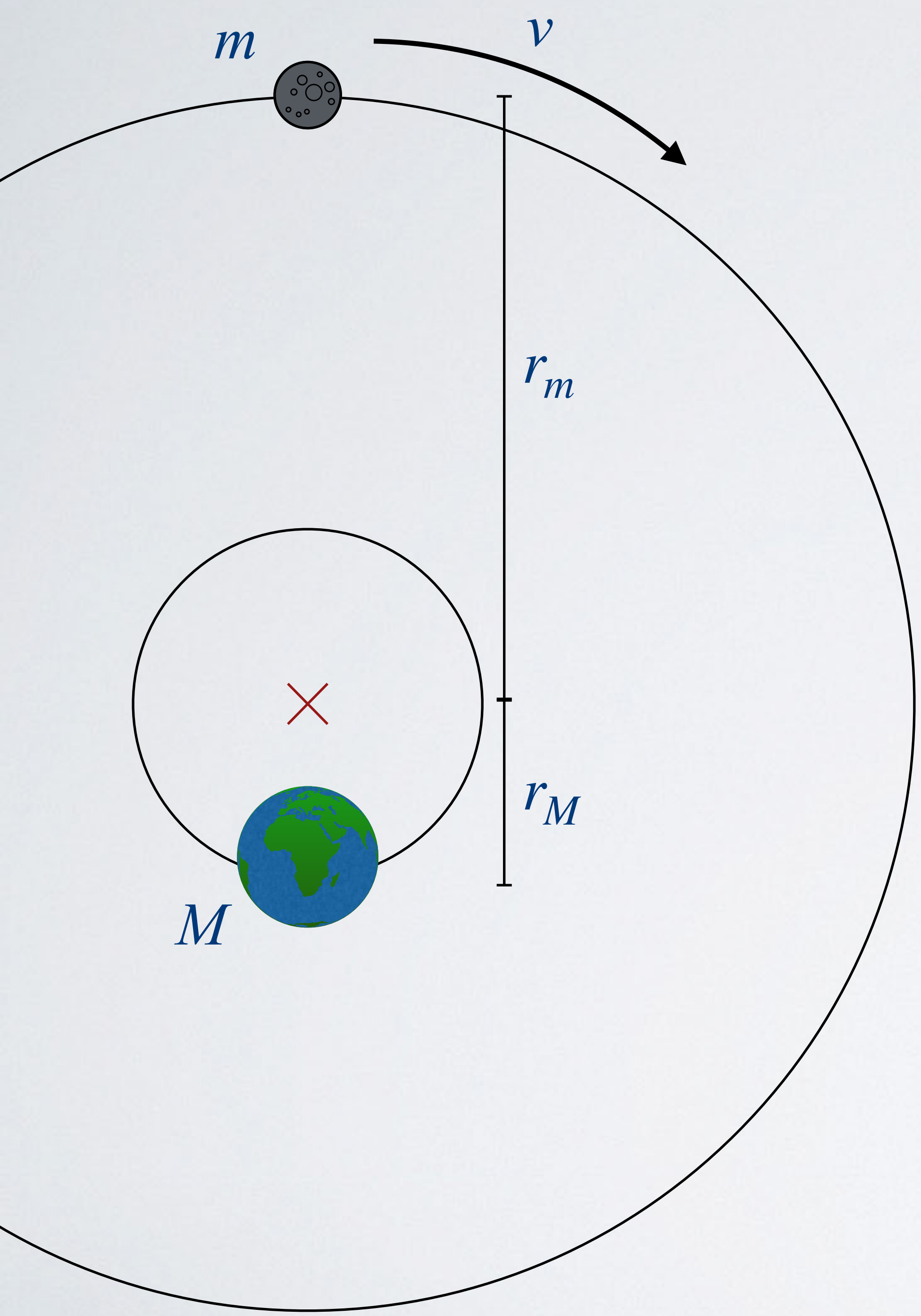

$$v^2 = \frac{GM}{r_m}\left(1 - 2m/M + \cdots\right)$$

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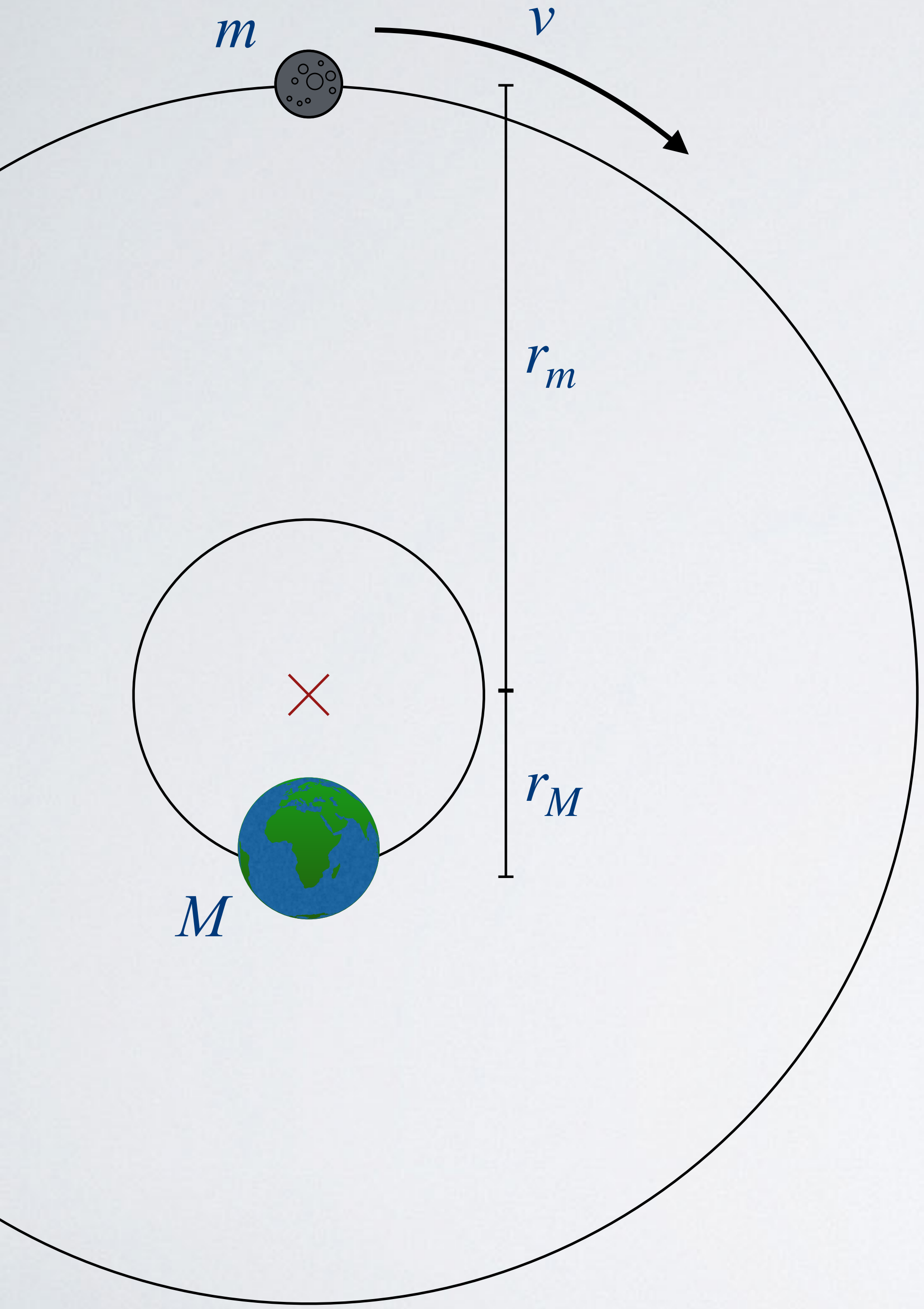
Define a radius in terms of observables



$$R_\Omega = \left(\frac{GM}{\Omega^2}\right)^{1/3}$$

$$v_\Omega^2 = \frac{GM}{R_\Omega}\left(1 - \frac{4}{3}\frac{m}{M} + \cdots\right)$$

Gauge invariants allow for useful comparisons

A consequence of the gravitational self-force for circular orbits of the Schwarzschild geometry

Steven Detweiler
Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, FL 32611-8440[*]
(Dated: April 22, 2008)

A small mass μ in orbit about a much more massive black hole m moves along a world line that deviates from a geodesic of the black hole geometry by $O(\mu/m)$. This deviation is said to be caused by the gravitational self-force of the metric perturbation h_{ab} from μ. For circular orbits about a non-rotating black hole we numerically calculate the $O(\mu/m)$ effects upon the orbital frequency and

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Two approaches for the gravitational self force in black hole spacetime: Comparison of numerical results

Norichika Sago[1], Leor Barack[1] and Steven Detweiler[2]
[1]*School of Mathematics, University of Southampton, Southampton, SO17 1BJ, United Kingdom*
[2]*Institute for Fundamental Theory, Department of Physics, University of Florida, Gainsville, FL 32611-8440*
(Dated: October 14, 2008)

Recently, two independent calculations have been presented of finite-mass ("self-force") effects on the orbit of a point mass around a Schwarzschild black hole. While both computations are based on the standard mode-sum method, they differ in several technical aspects, which makes comparison between their results difficult—but also interesting. Barack and Sago [Phys. Rev. D **75**, 064021

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2. Keeps you honest
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   - What does *gauge invariant* mean?

3. EOB calibration

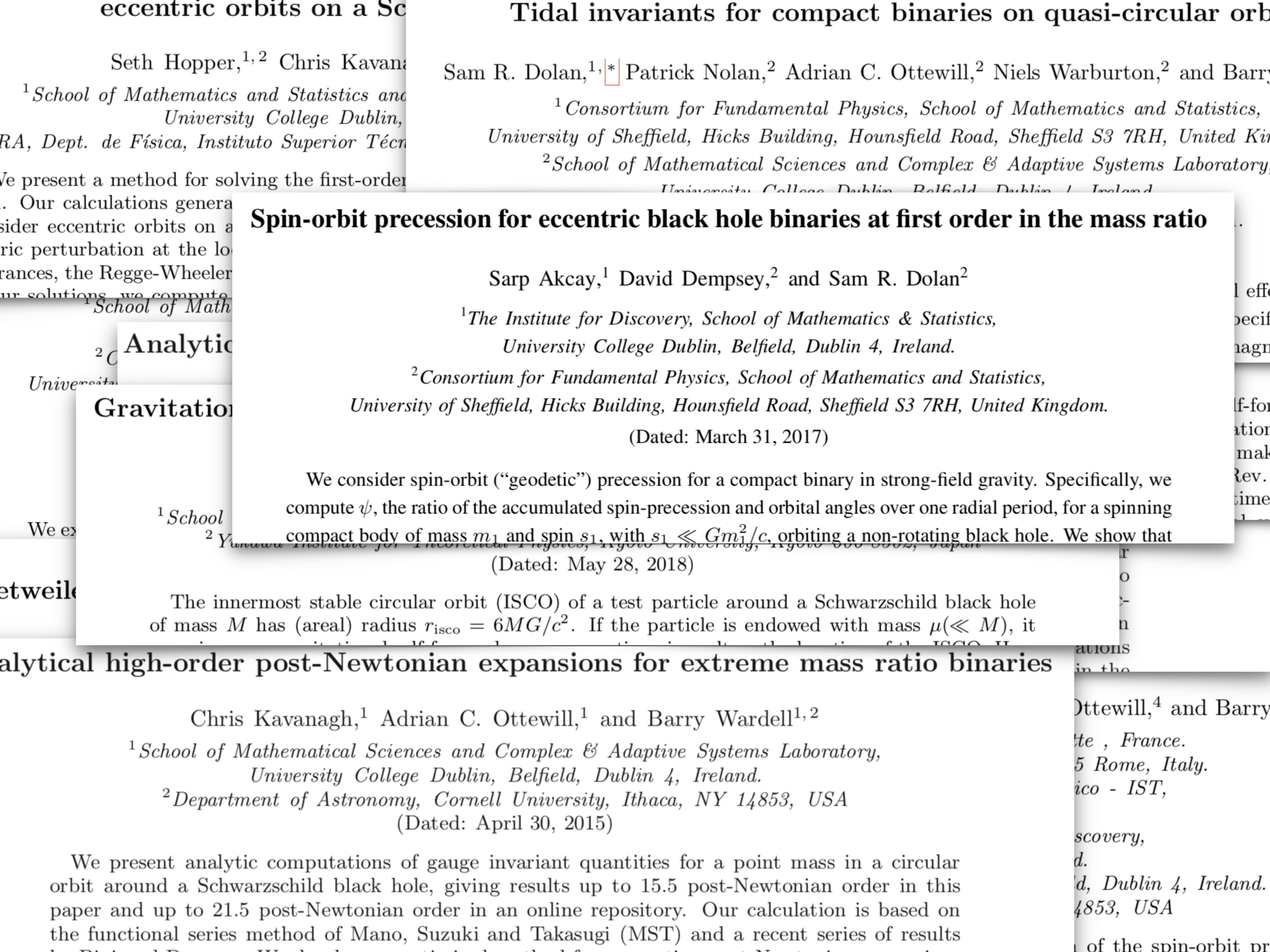
The era of gauge invariants has been a time of plenty

Gravitational self-torque and spin precession in compact binaries

Sam R. Dolan,[1,*] Niels Warburton,[2] Abraham I. Harte,[3] Alexandre Le Tiec,[4,5] Barry Wardell,[2,6] and Leor Barack[7]

[1]*Consortium for Fundamental Physics, School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom.*
[2]*School of Mathematical Sciences and Complex & Adaptive Systems Laboratory, University College Dublin, Belfield, Dublin 4, Ireland.*
[3]*Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Am Mühlenberg 1, 14476 Golm, Germany.*
[4]*Maryland Center for Fundamental Physics & Joint Space-Science Institute, Department of Physics, University of Maryland, College Park, MD 20742, USA.*
[5]*Laboratoire Univers et Théories (LUTh), Observatoire de Paris, CNRS, Université Paris Diderot, 5 place Jules Janssen, 92190 Meudon, France.*
[6]*Department of Astronomy, Cornell University, Ithaca, NY 14853, USA.*
[7]*School of Mathematics, University of Southampton, Southampton SO17 1BJ, United Kingdom.*
(Dated: March 10, 2014)

We calculate the effect of self-interaction on the "geodetic" spin precession of a compact body in a strong-field orbit around a black hole. Specifically, we consider the spin precession angle ψ per radian of orbital revolution



Tidal invariants for compact binaries on quasi-circular orb

Sam R. Dolan,[1,*] Patrick Nolan,[2] Adrian C. Ottewill,[2] Niels Warburton,[2] and Barry

[1]Consortium for Fundamental Physics, School of Mathematics and Statistics,
University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Ki
[2]School of Mathematical Sciences and Complex & Adaptive Systems Laboratory,

eccentric orbits on a Sc

Seth Hopper,[1,2] Chris Kavana

[1]School of Mathematics and Statistics and
University College Dublin,
RA, Dept. de Física, Instituto Superior Técn

Ve present a method for solving the first-order
. Our calculations genera
sider eccentric orbits on a
ric perturbation at the lo
rances, the Regge-Wheeler

Spin-orbit precession for eccentric black hole binaries at first order in the mass ratio

Sarp Akcay,[1] David Dempsey,[2] and Sam R. Dolan[2]

[1]The Institute for Discovery, School of Mathematics & Statistics,
University College Dublin, Belfield, Dublin 4, Ireland.
[2]Consortium for Fundamental Physics, School of Mathematics and Statistics,
University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom.

(Dated: March 31, 2017)

We consider spin-orbit ("geodetic") precession for a compact binary in strong-field gravity. Specifically, we compute ψ, the ratio of the accumulated spin-precession and orbital angles over one radial period, for a spinning compact body of mass m_1 and spin s_1, with $s_1 \ll Gm_1^2/c$, orbiting a non-rotating black hole. We show that

Analytic

Gravitation

(Dated: May 28, 2018)

The innermost stable circular orbit (ISCO) of a test particle around a Schwarzschild black hole of mass M has (areal) radius $r_{\rm isco} = 6MG/c^2$. If the particle is endowed with mass $\mu (\ll M)$, it

alytical high-order post-Newtonian expansions for extreme mass ratio binaries

Chris Kavanagh,[1] Adrian C. Ottewill,[1] and Barry Wardell[1,2]

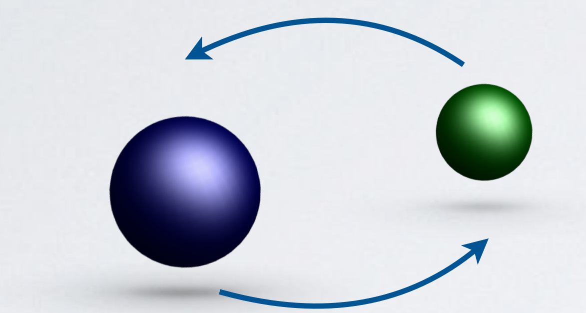
[1]School of Mathematical Sciences and Complex & Adaptive Systems Laboratory,
University College Dublin, Belfield, Dublin 4, Ireland.
[2]Department of Astronomy, Cornell University, Ithaca, NY 14853, USA

(Dated: April 30, 2015)

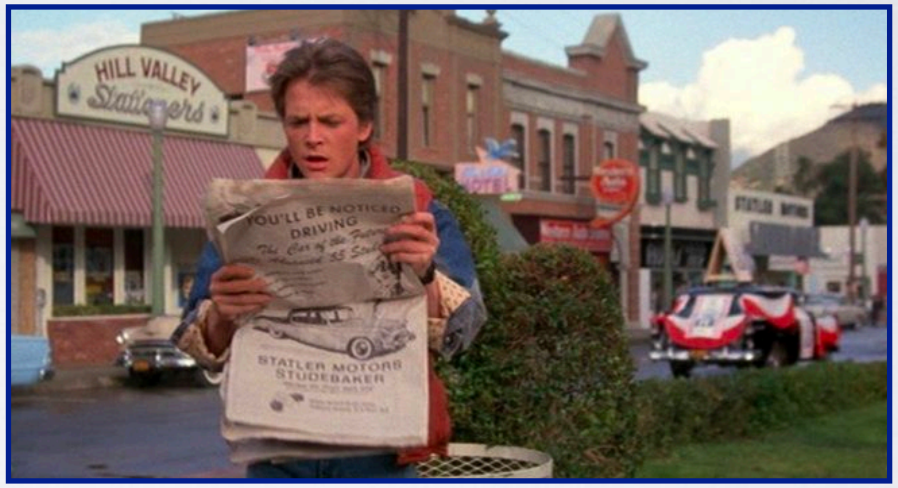
We present analytic computations of gauge invariant quantities for a point mass in a circular orbit around a Schwarzschild black hole, giving results up to 15.5 post-Newtonian order in this paper and up to 21.5 post-Newtonian order in an online repository. Our calculation is based on the functional series method of Mano, Suzuki and Takasugi (MST) and a recent series of results

Outline

Why we're here



Yesterday



Today

Tomorrow

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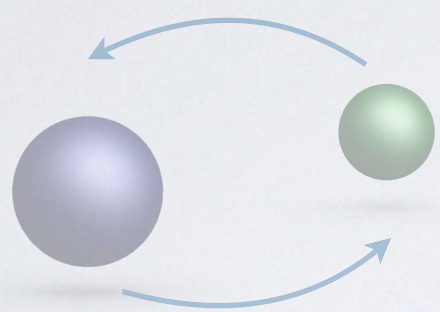

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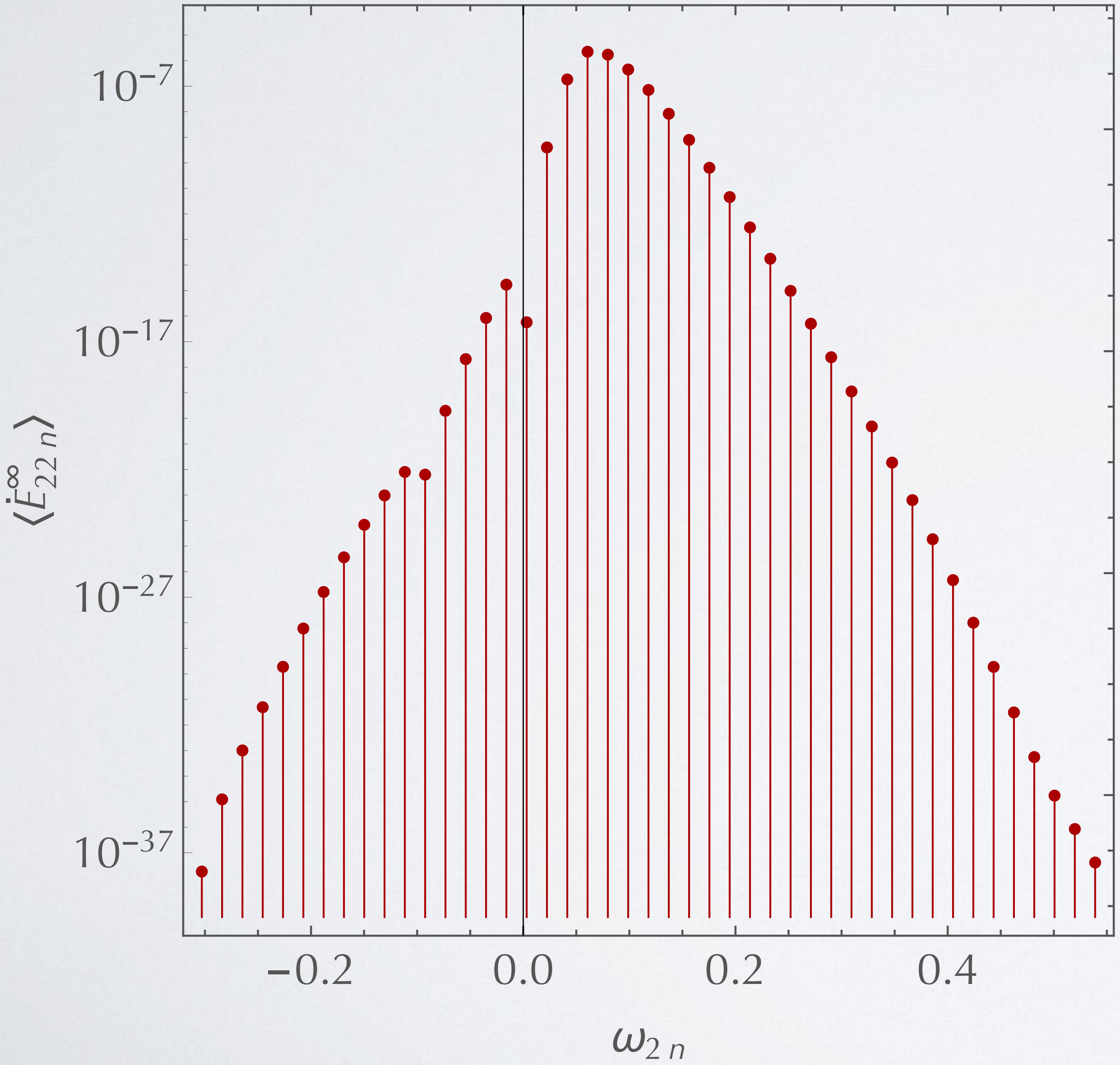
Background geodesics + frequency domain + mode-sum regularization codes have long been the standard



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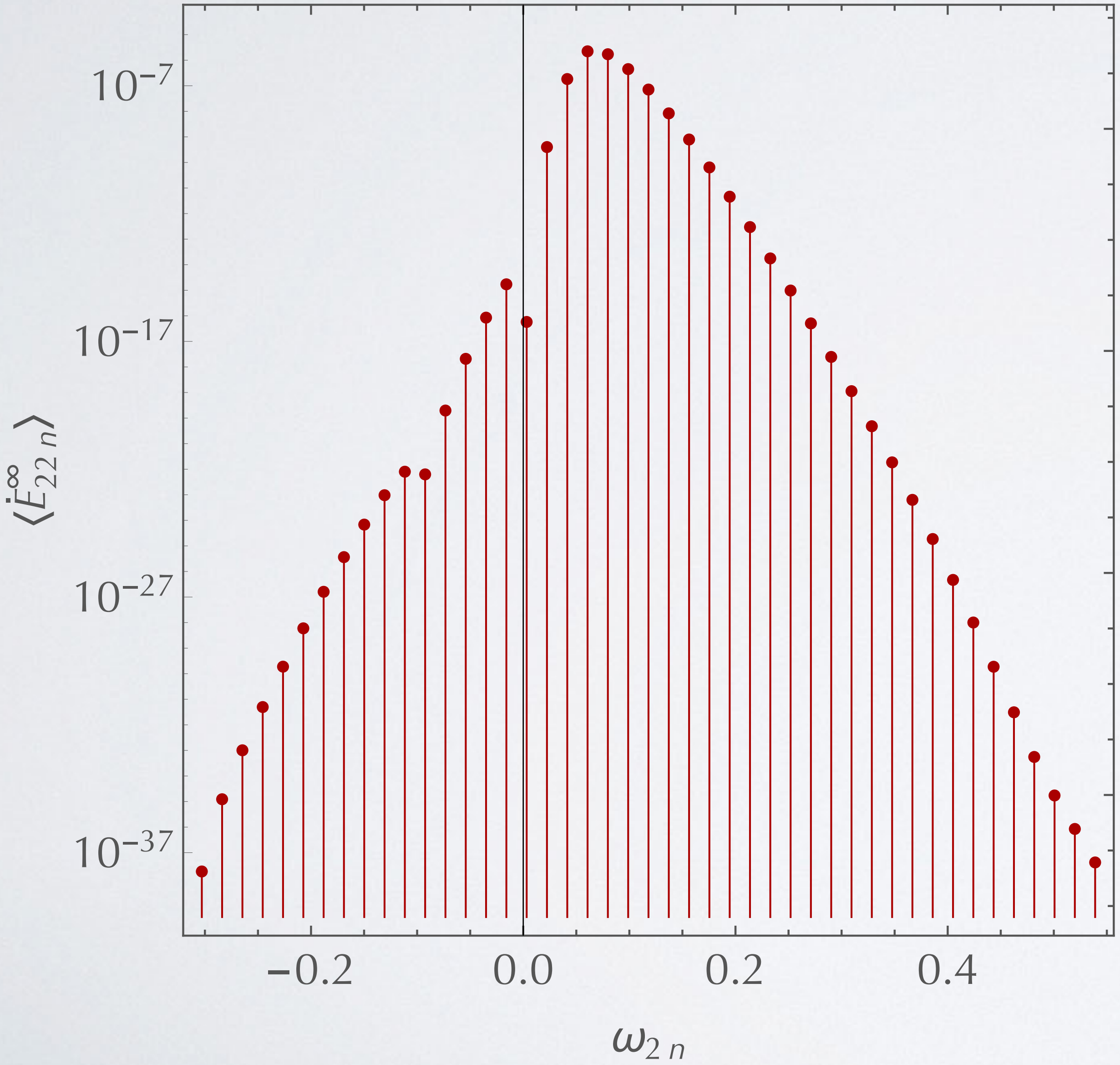

- ODEs

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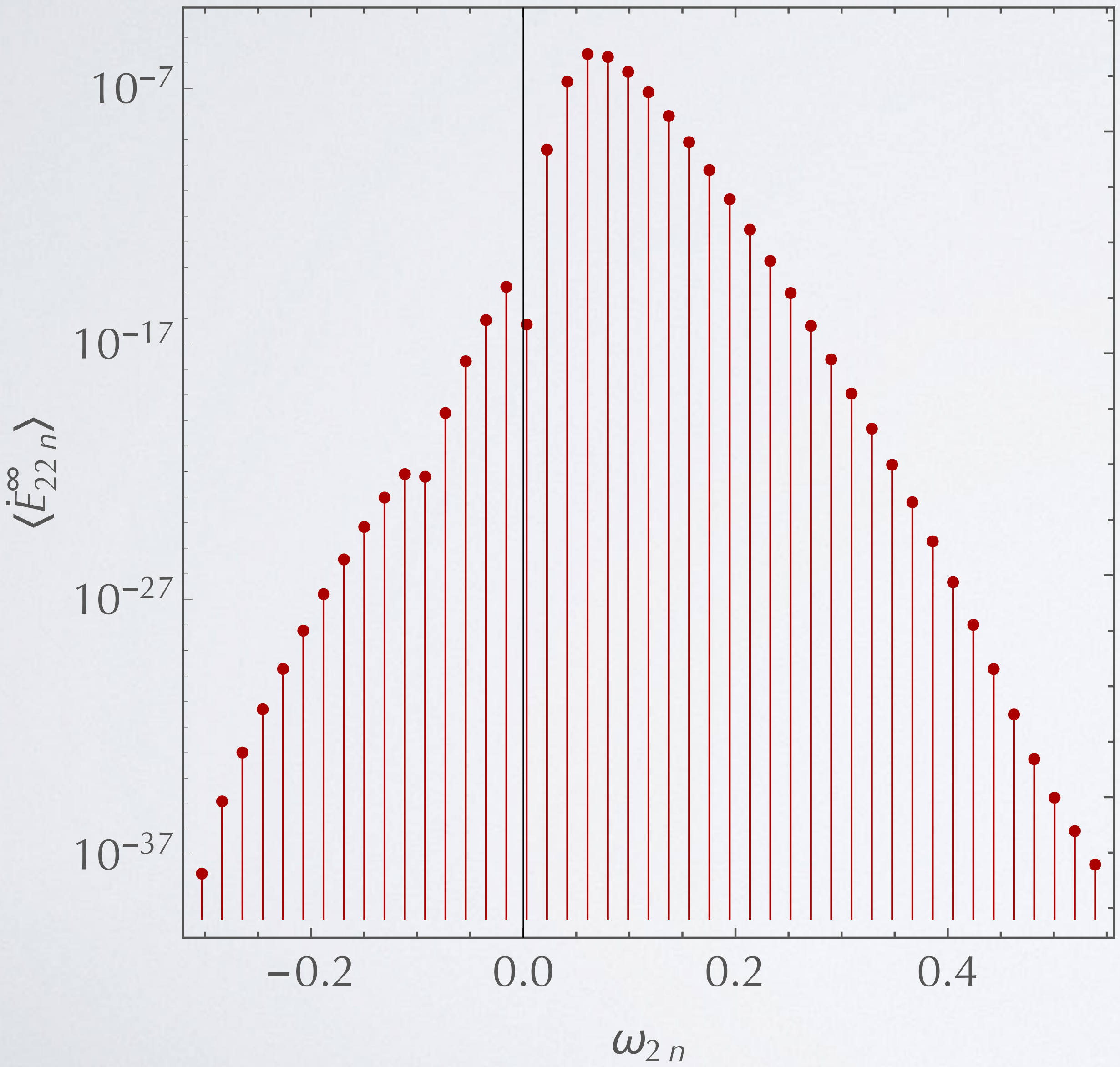

- ODEs
- Discrete spectra

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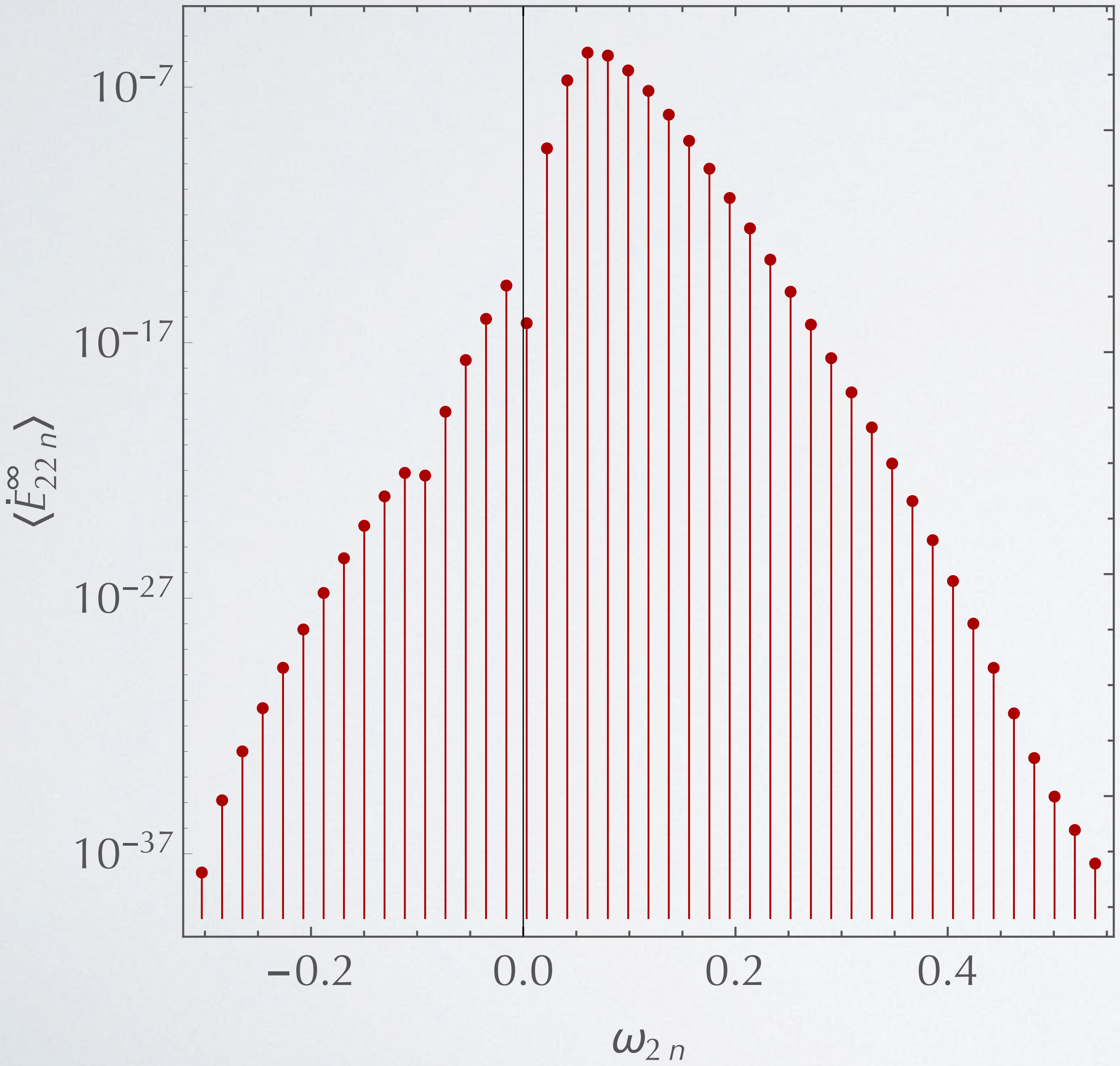

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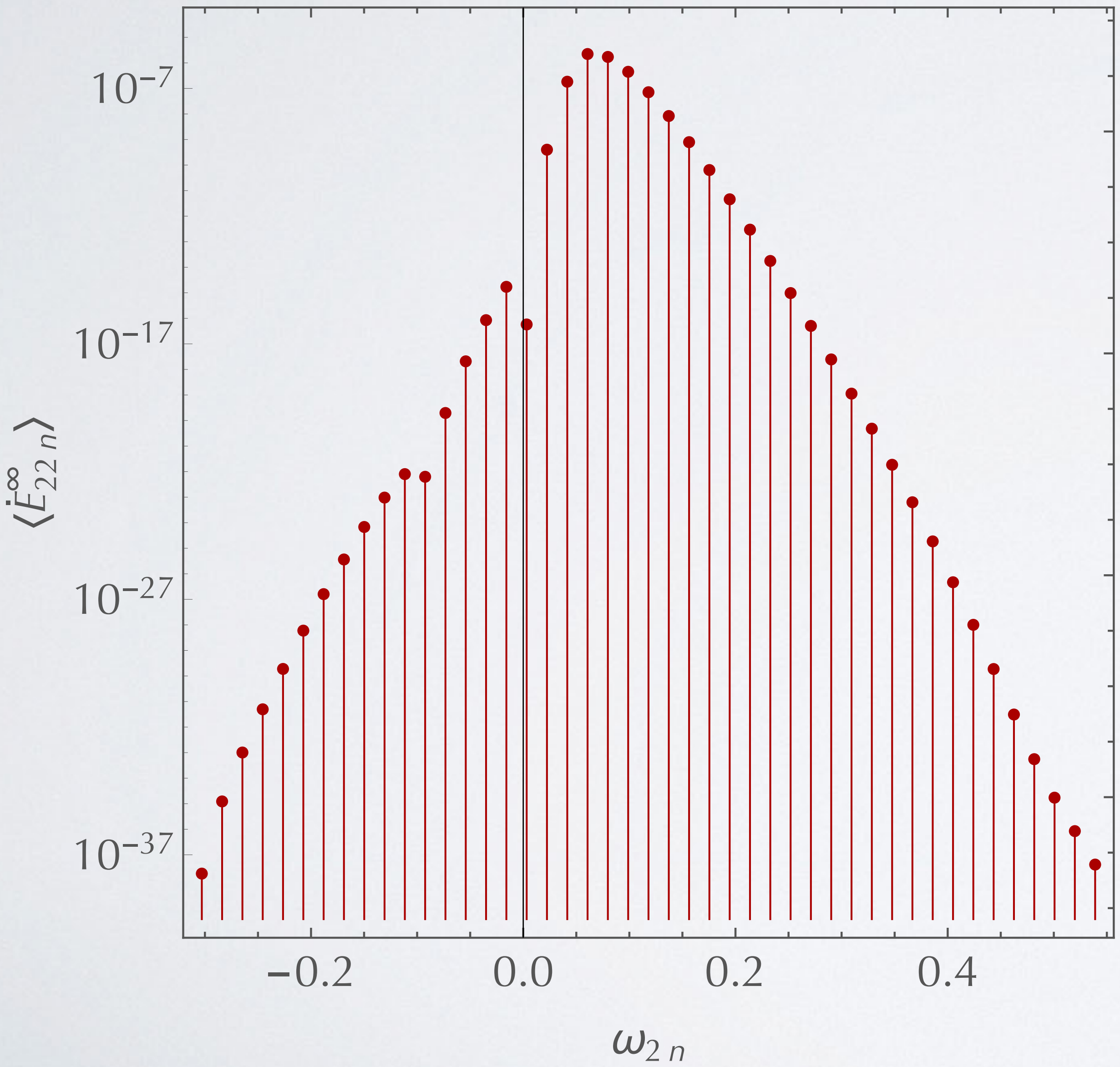

- ODEs
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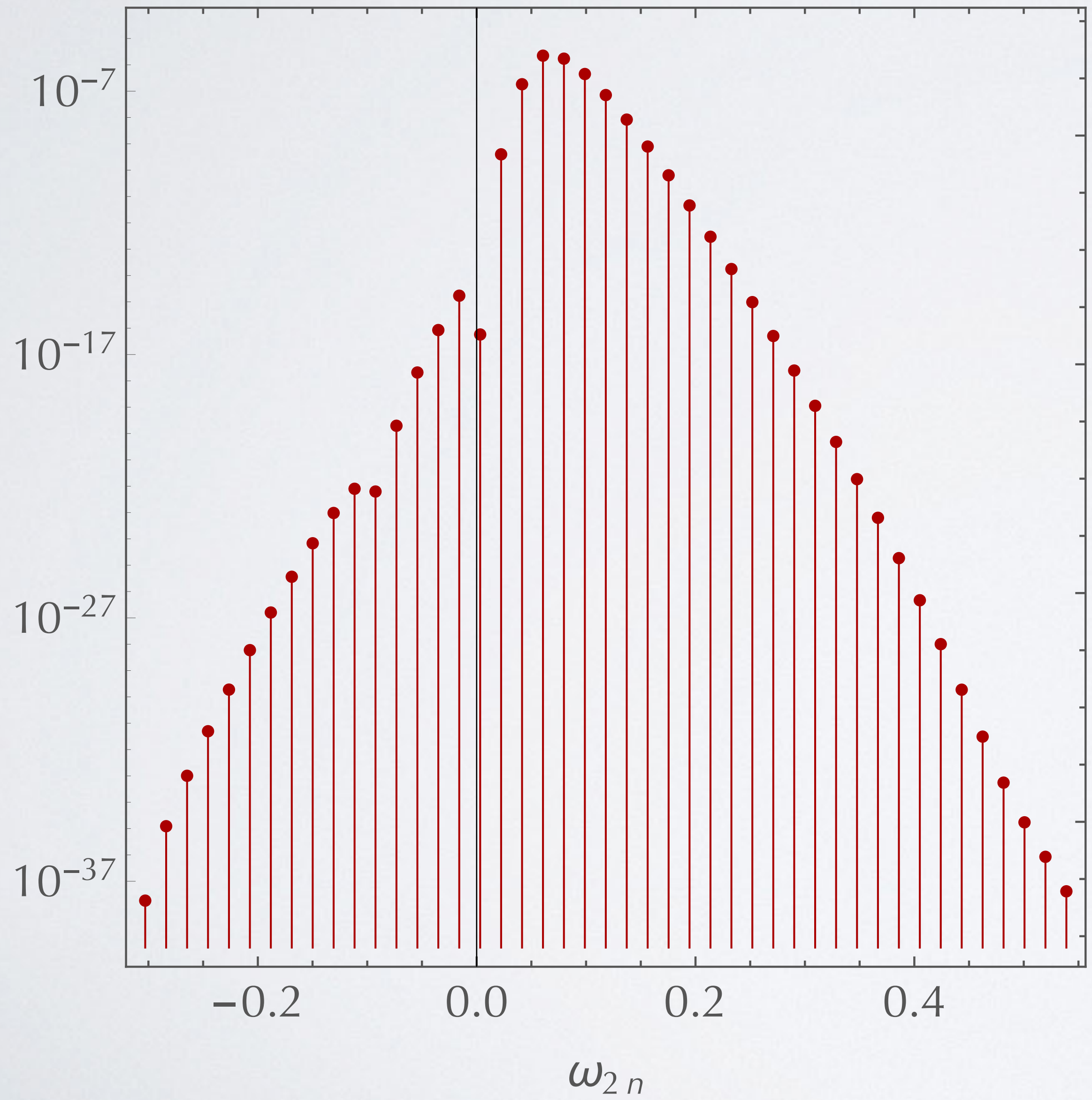

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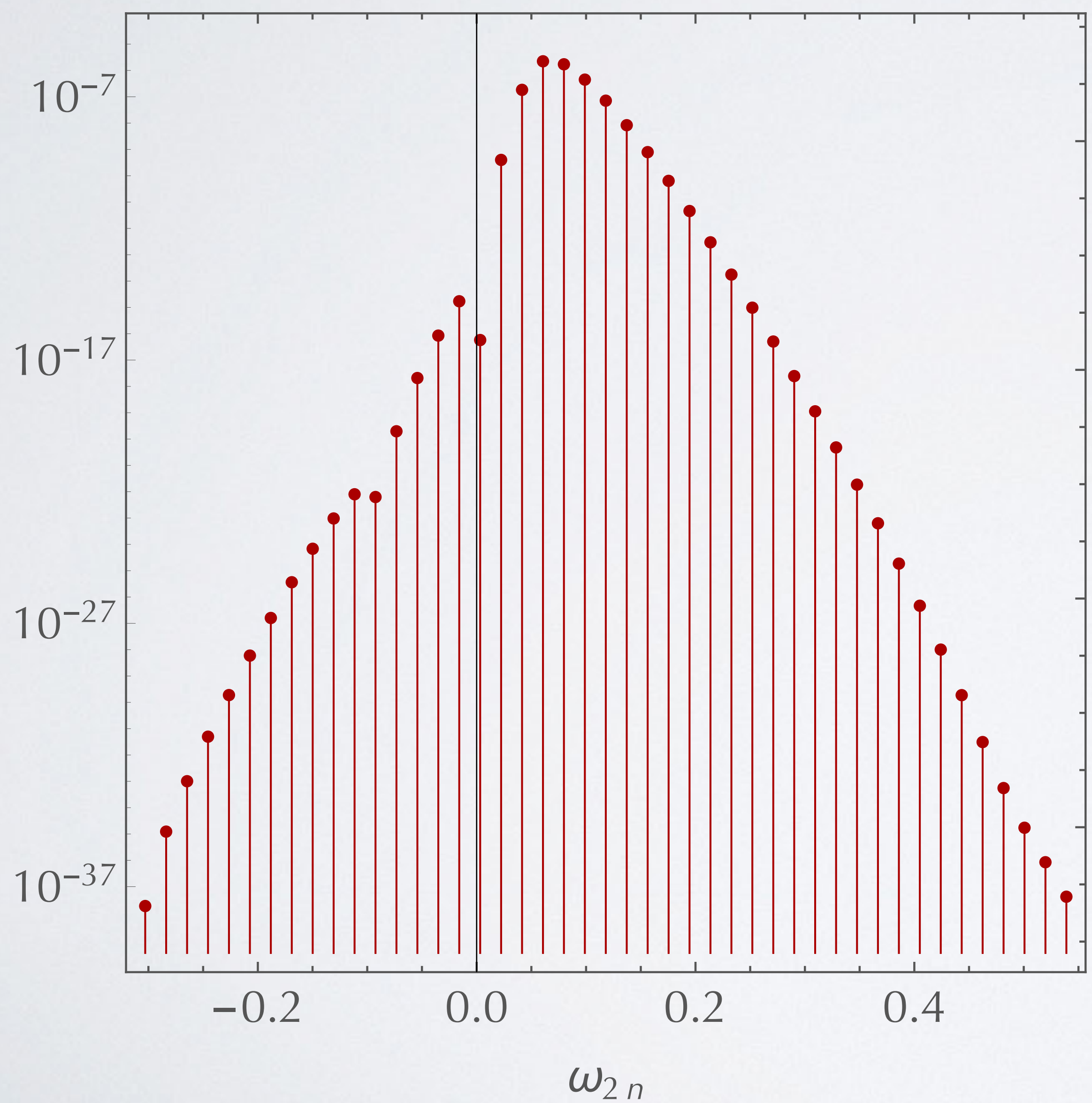

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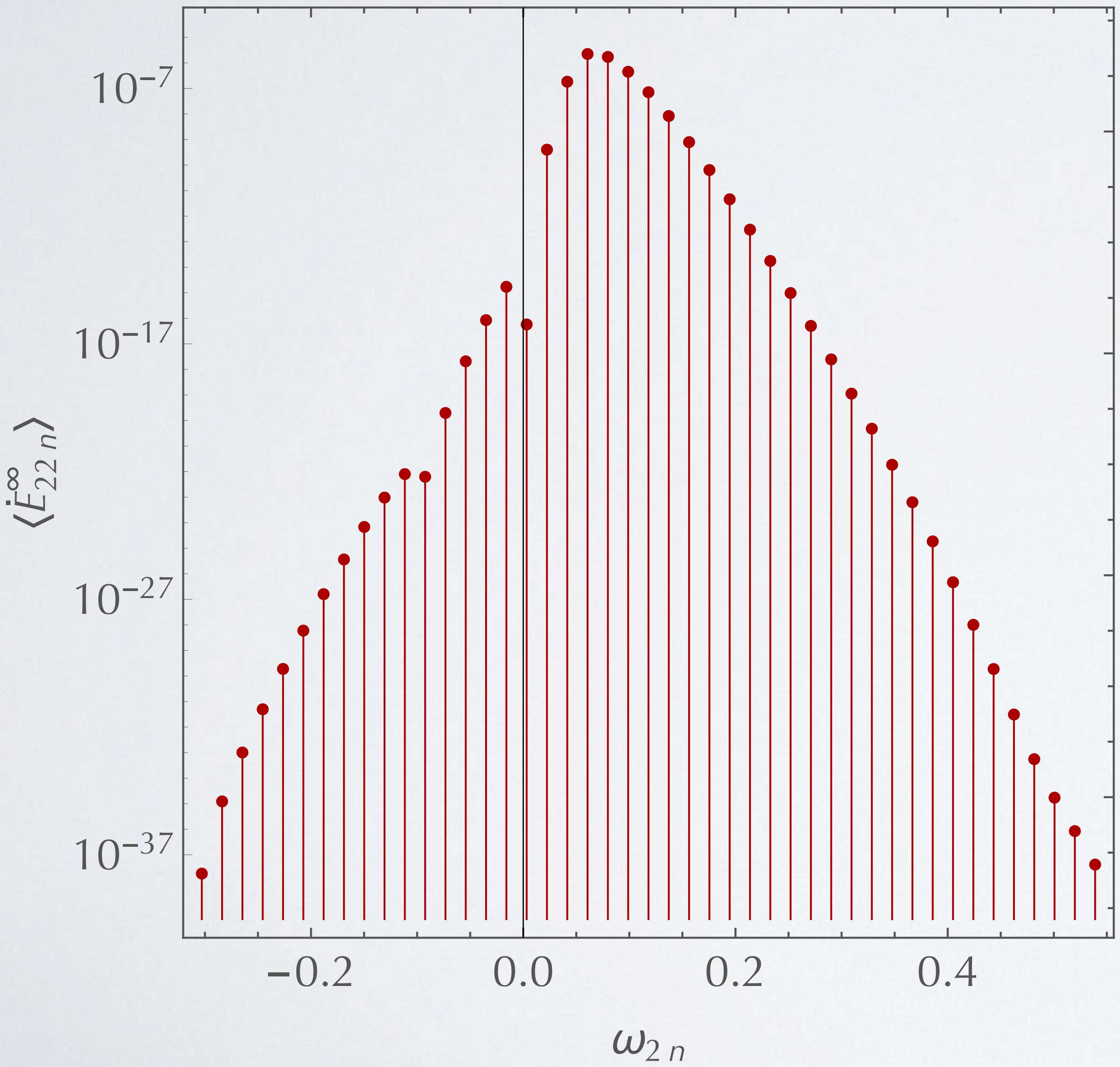

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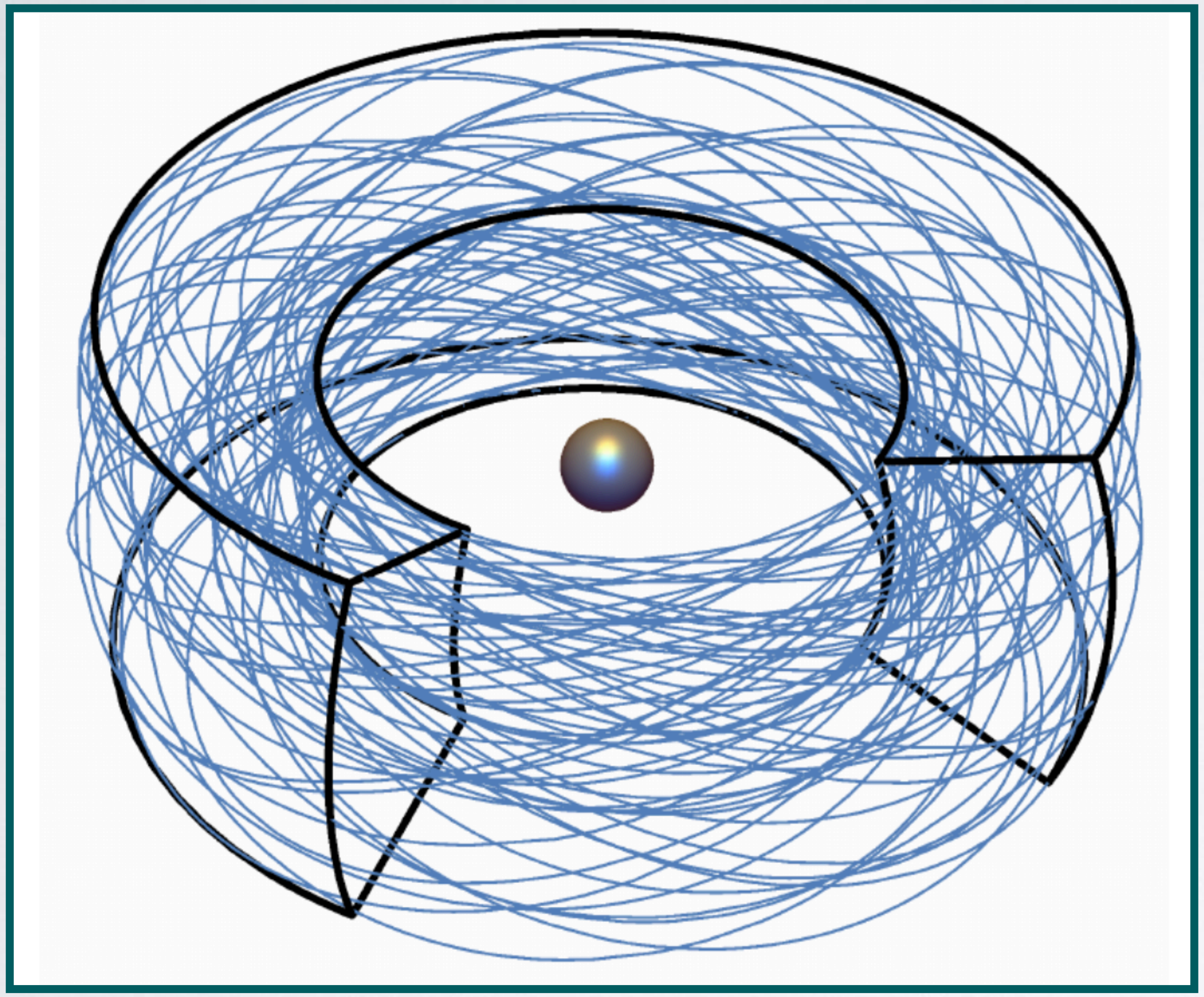
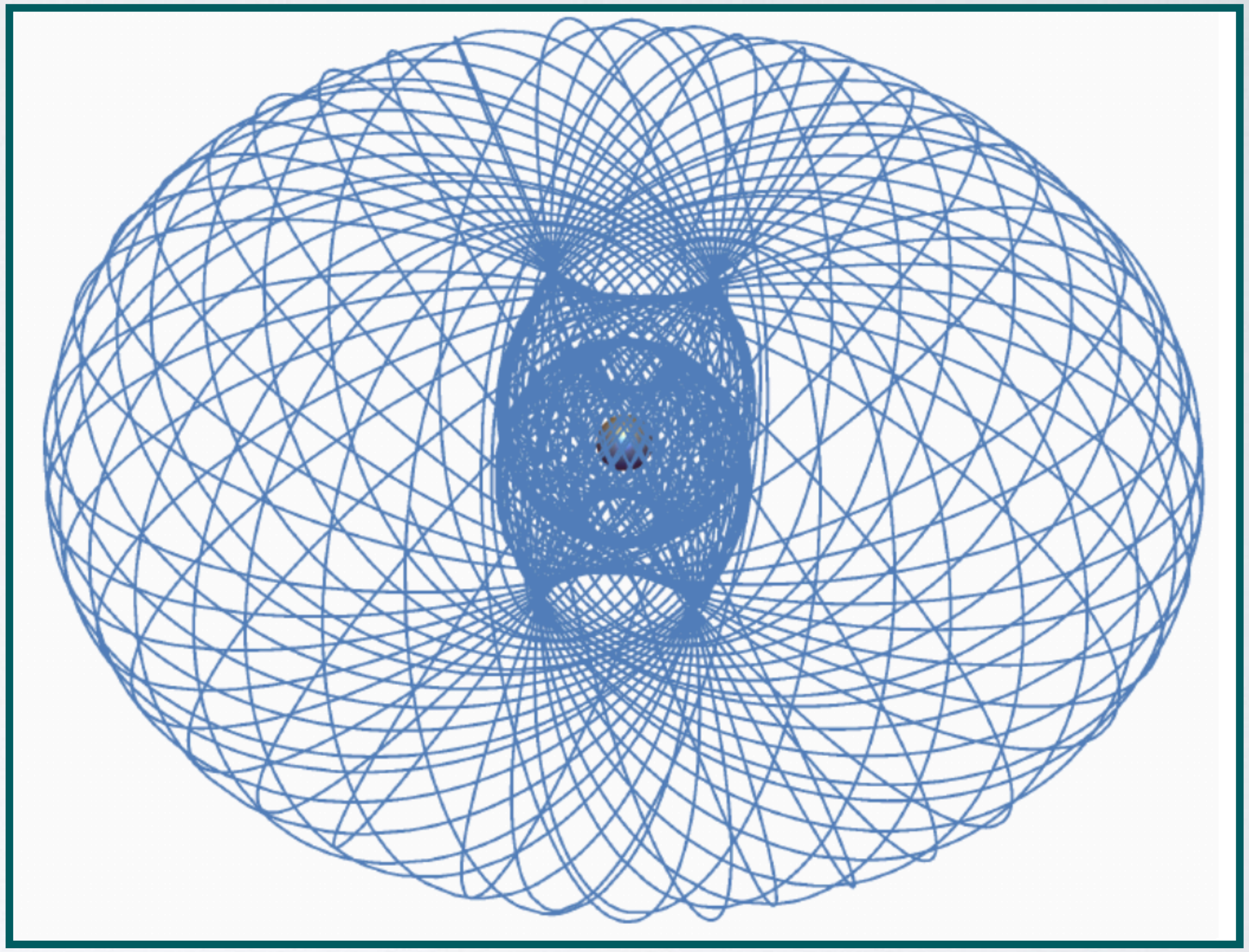
- Evolve with osculating geodesics

Even with these benefits, generic orbits on Kerr are challenging

Barack & Pound, 2018



Generic, ergodic orbit



Resonant orbit

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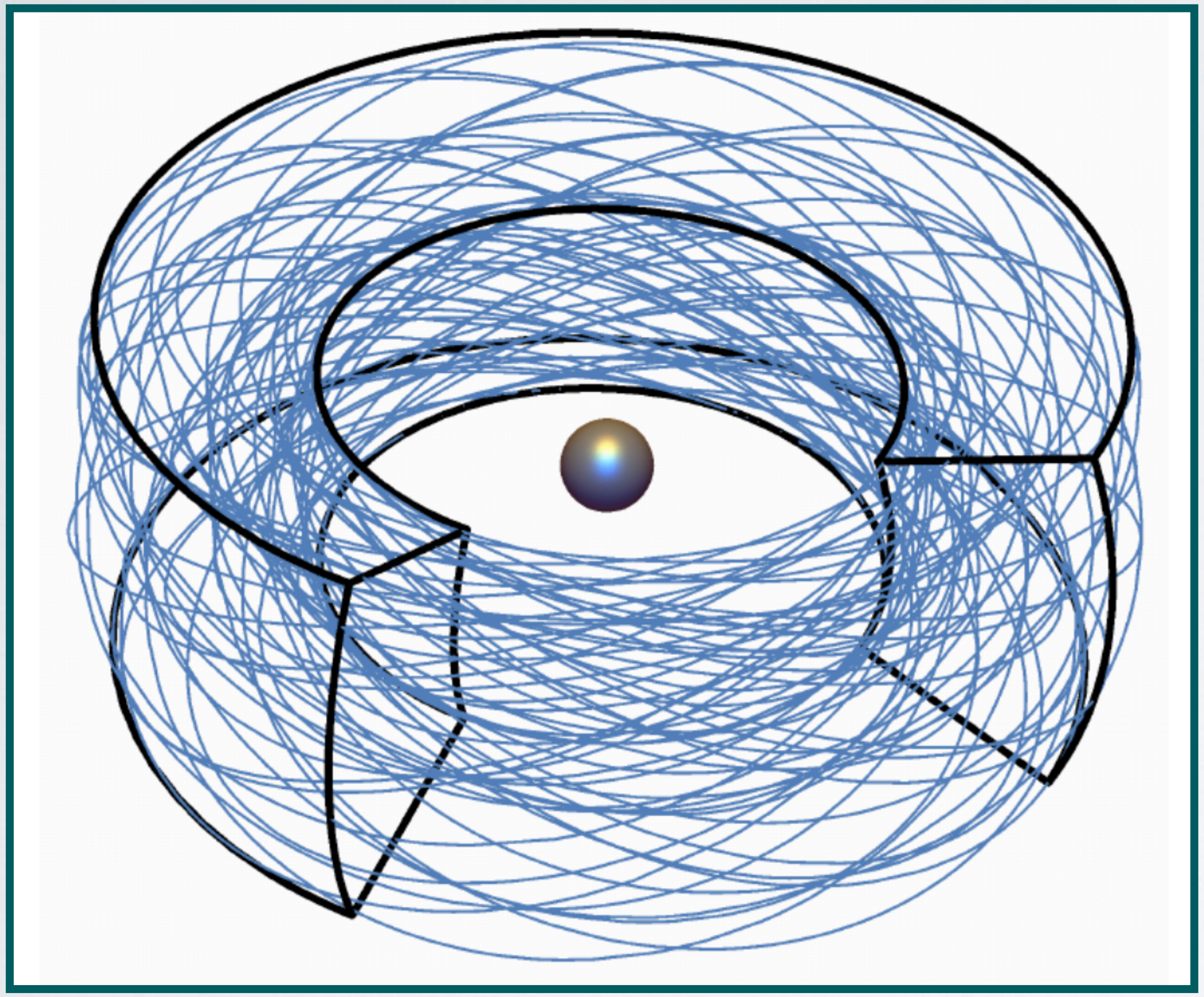

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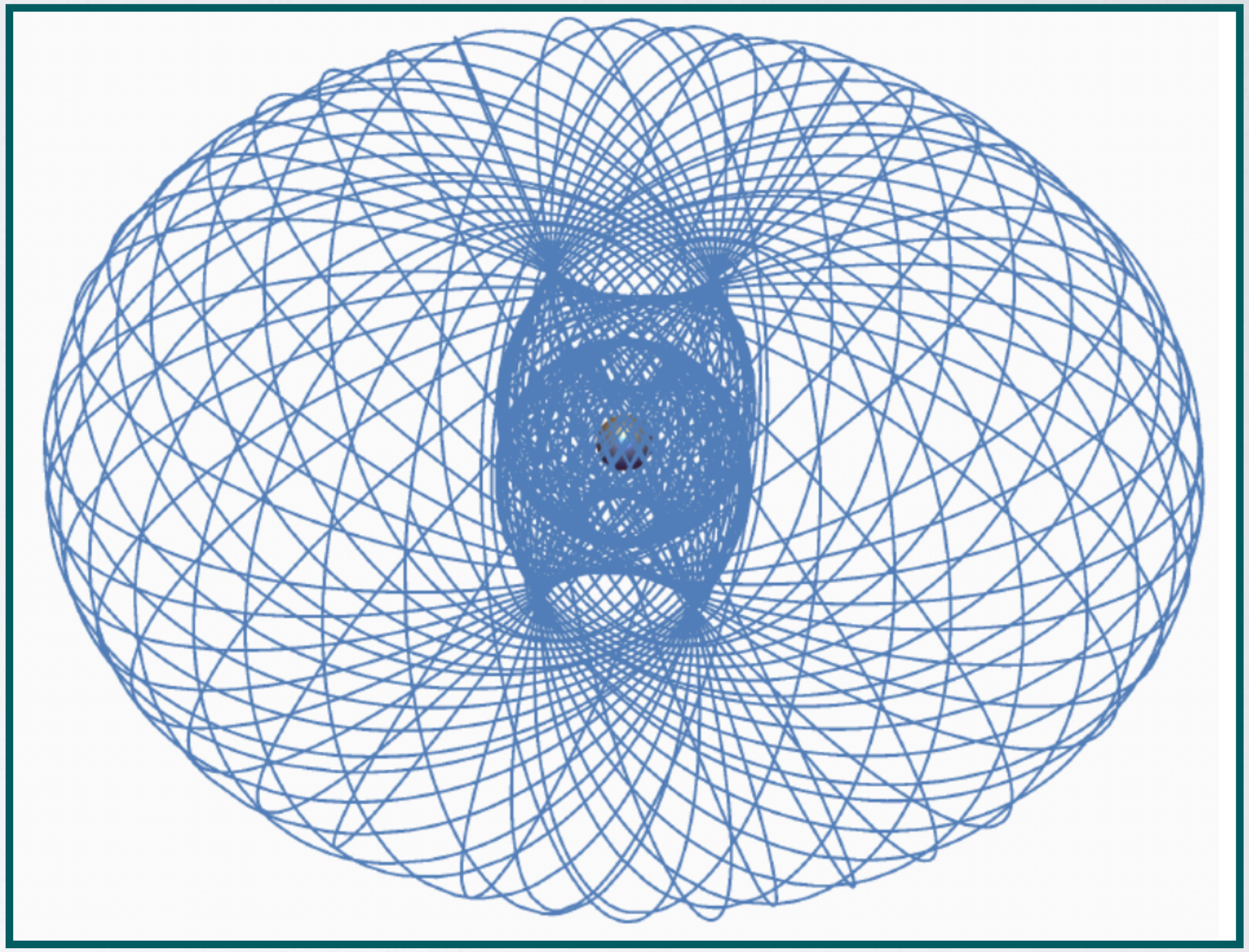

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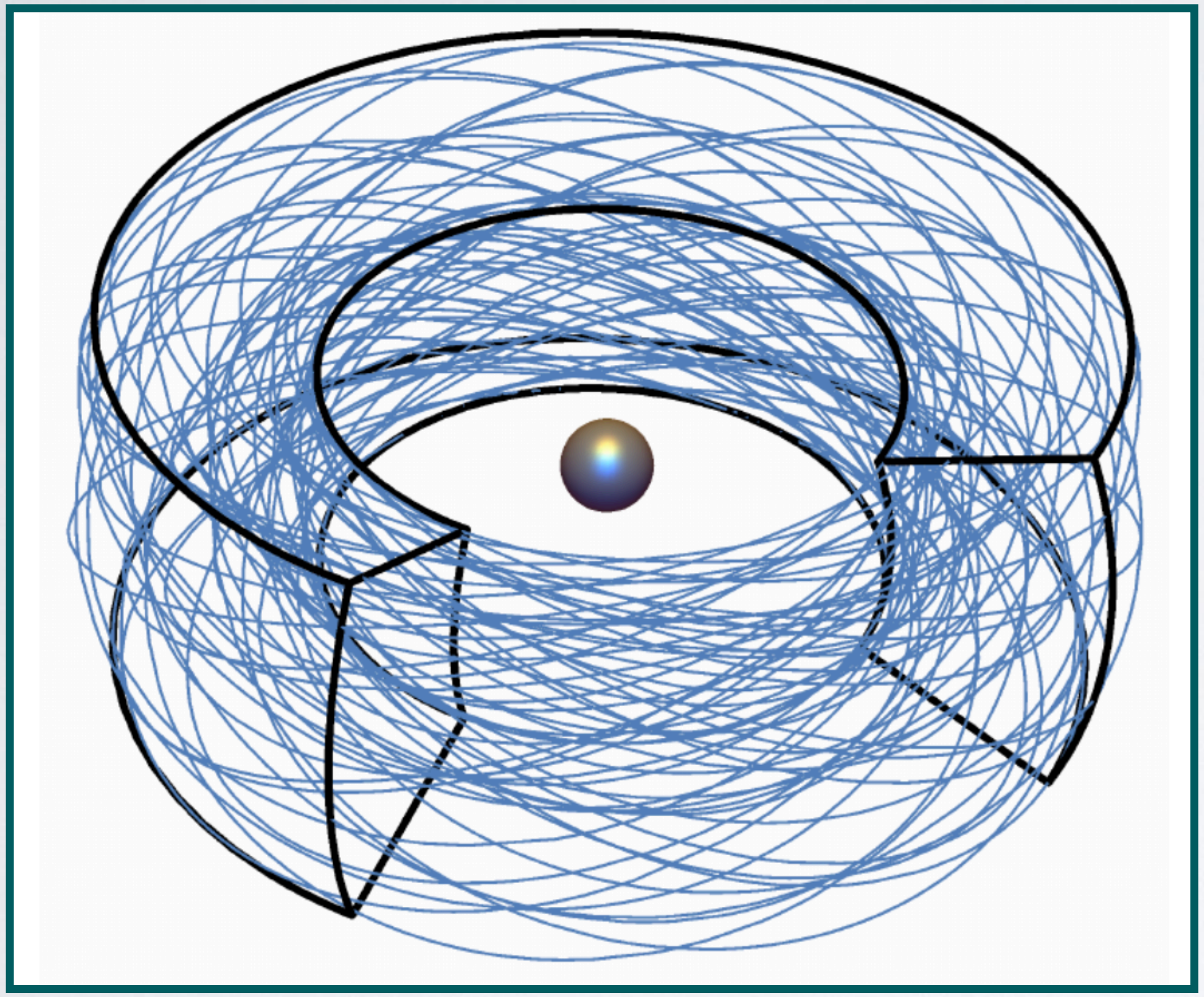

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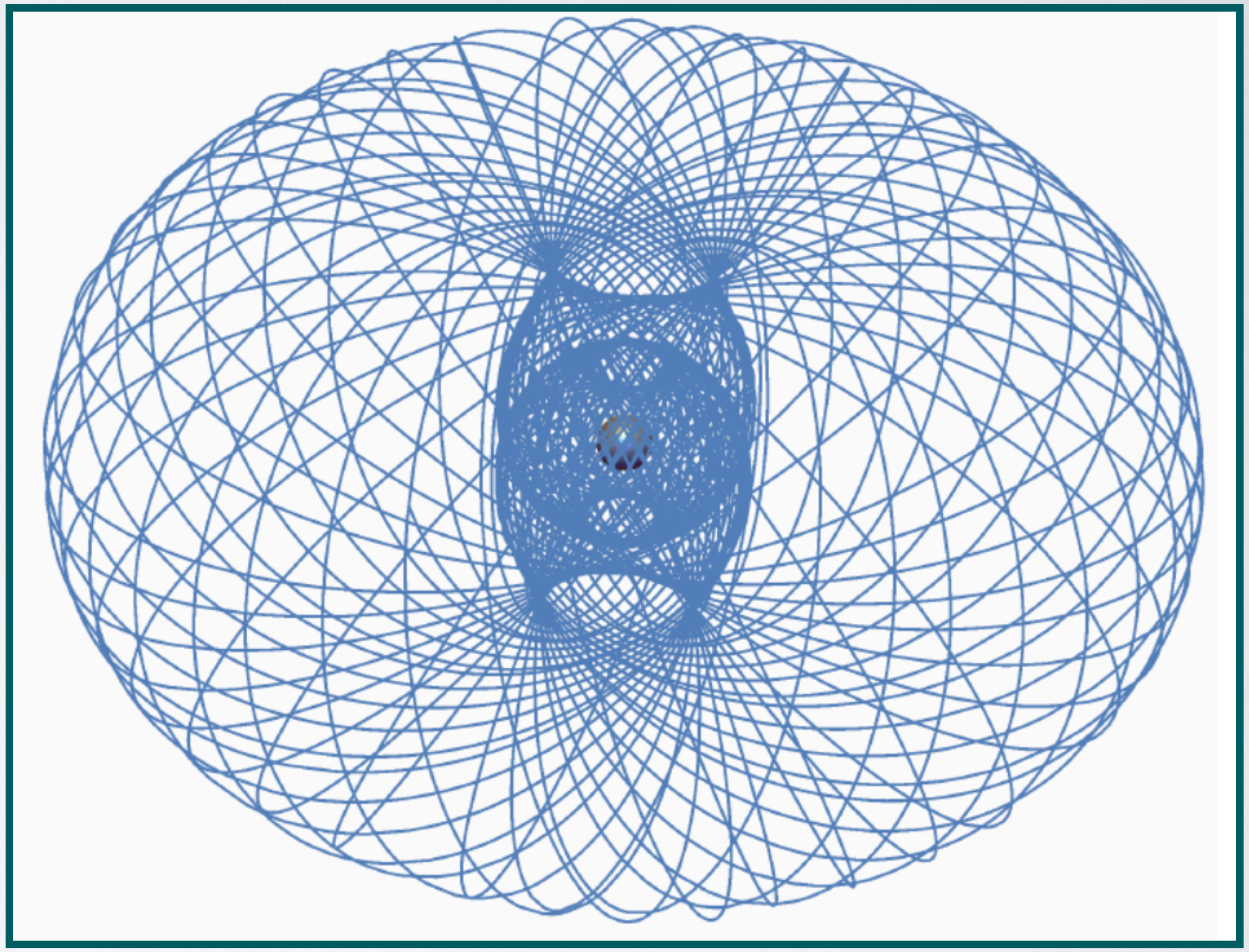
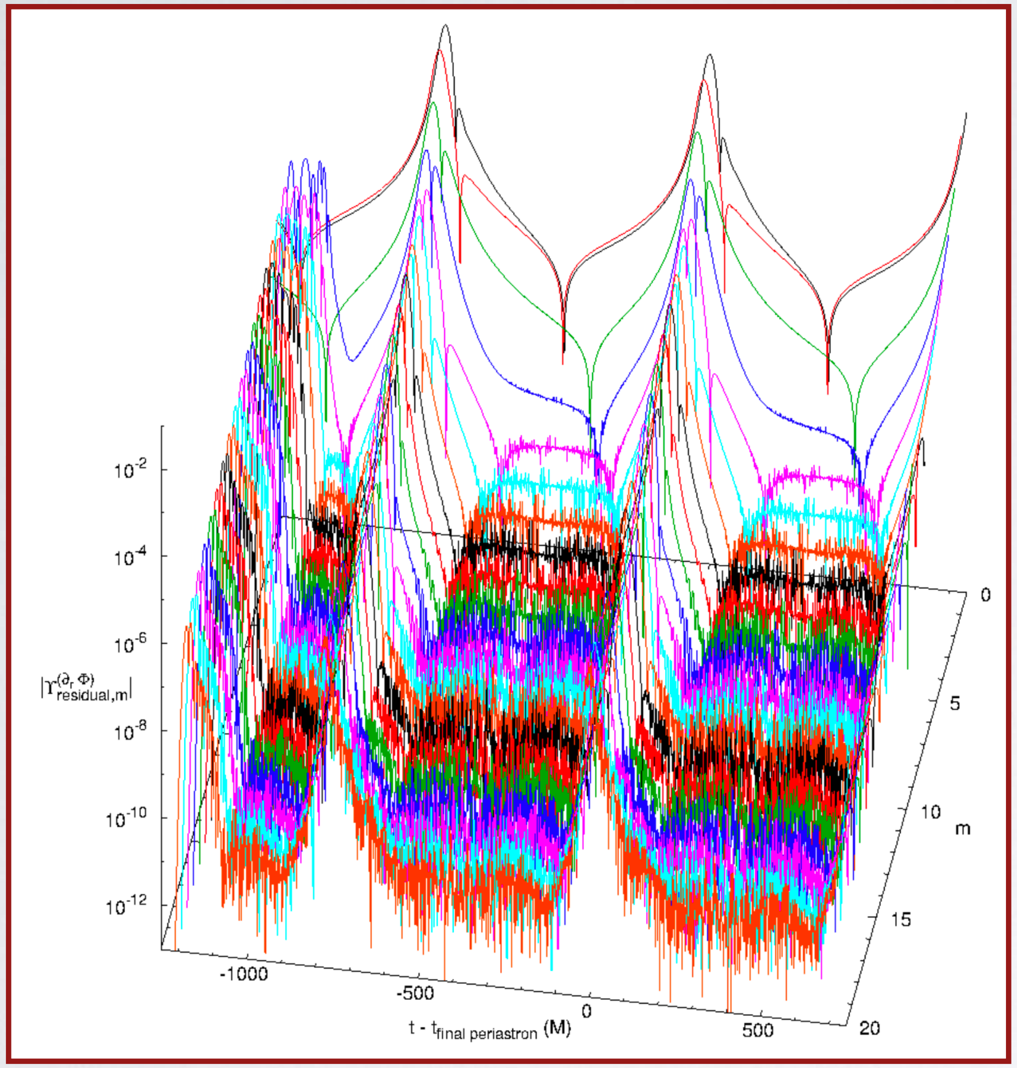


Resonant orbit

Maarten van de Meent - Next

Zachary Nasipak - Today

Time domain codes can (in theory) perform self-consistent evolutions



Thornburg, Wardell, 2017

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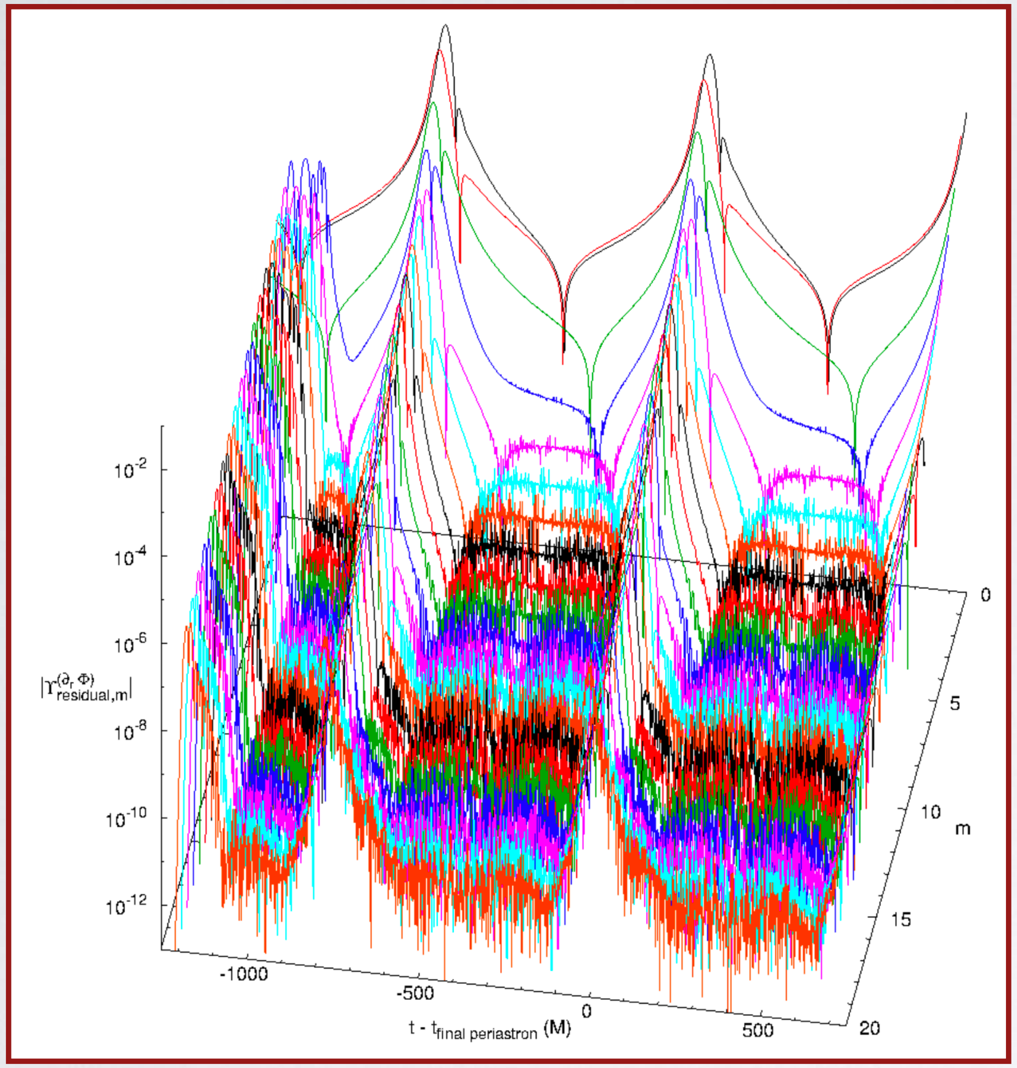

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Jonathan Thornburg - Today

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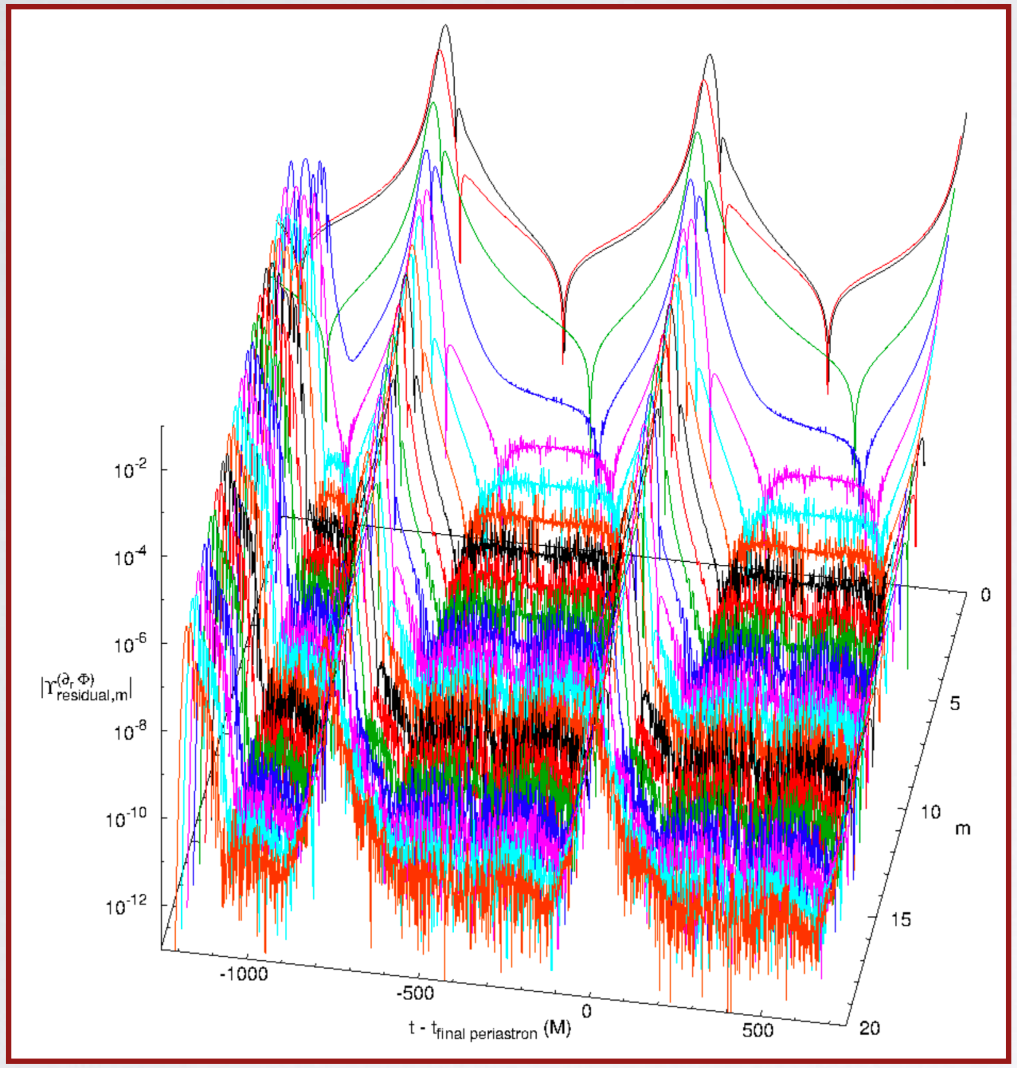

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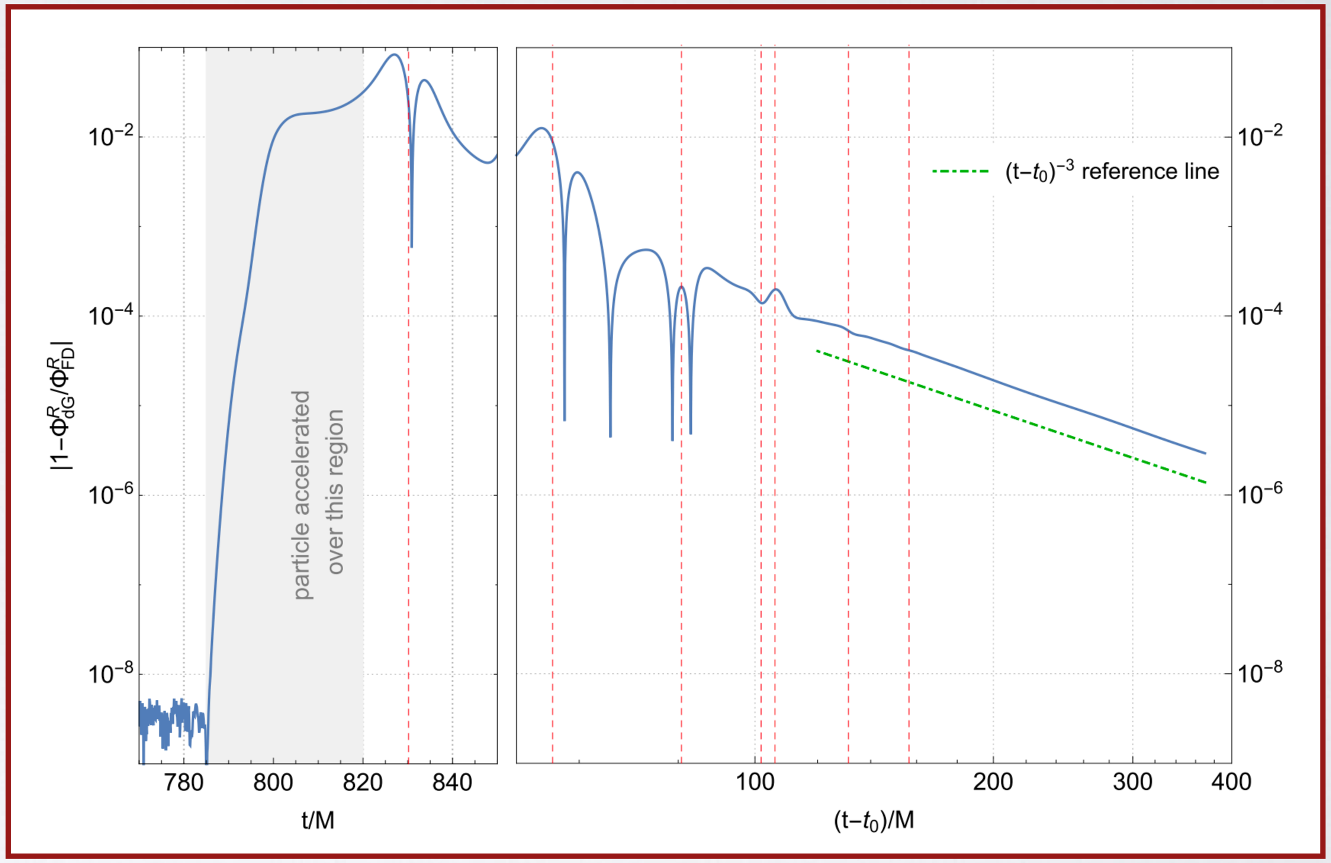

Heffernan, et al., 2017

Jonathan Thornburg - Today

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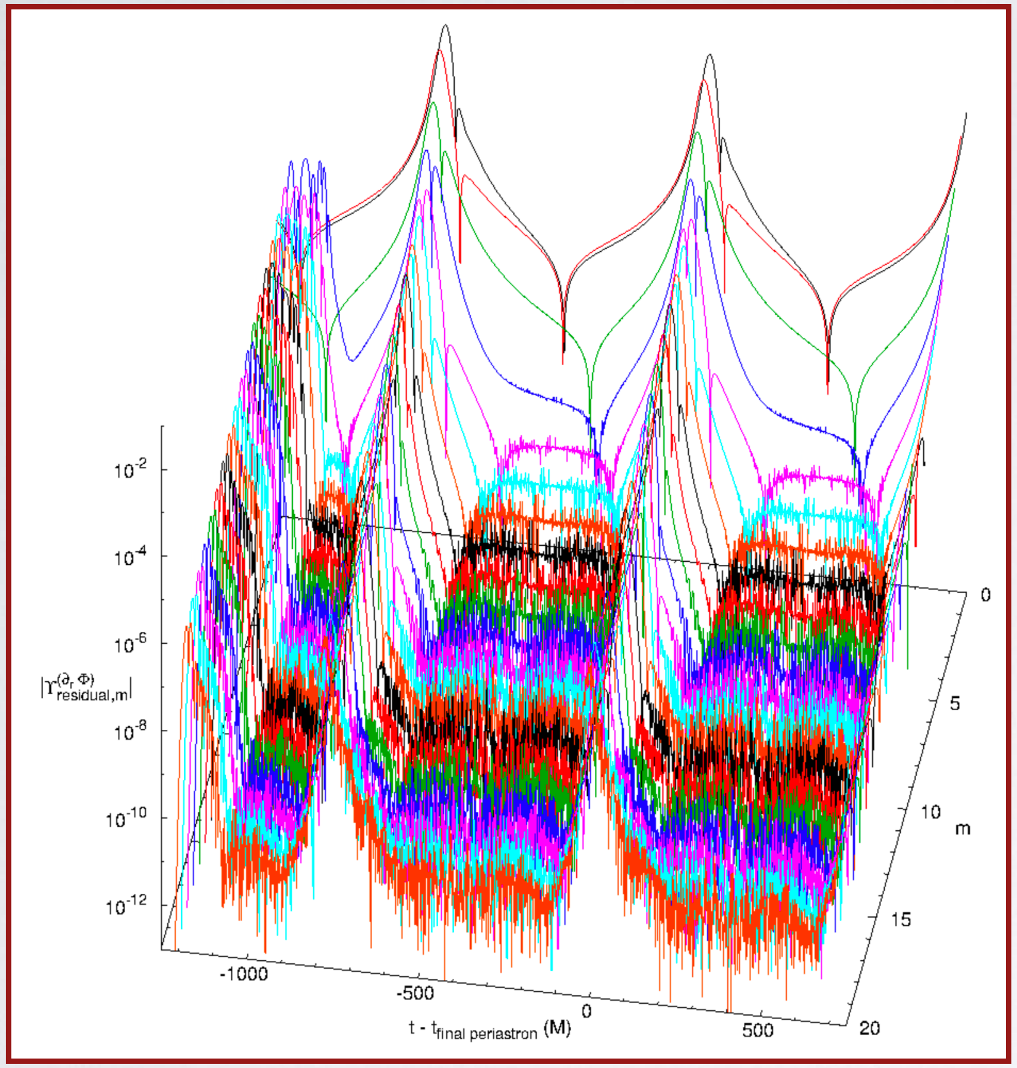
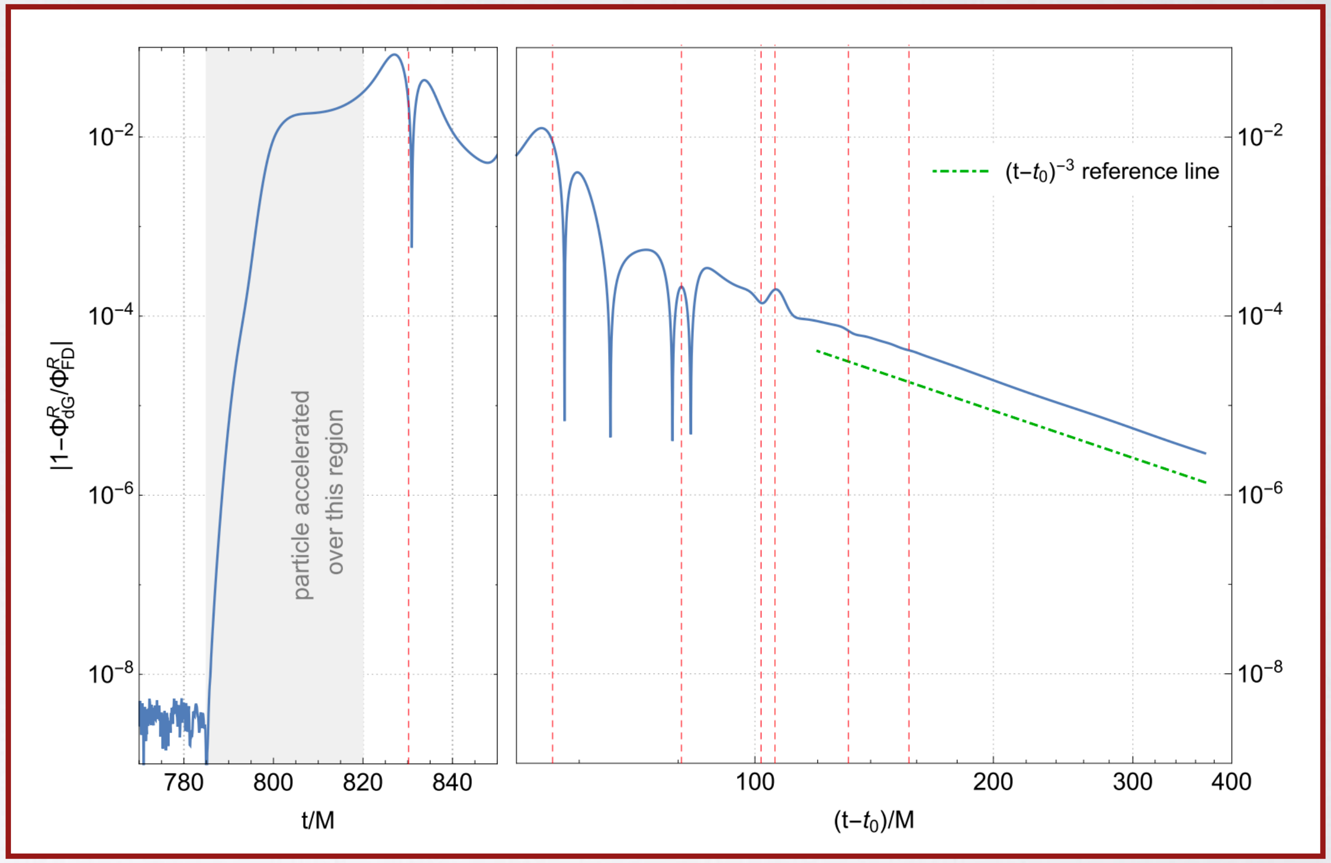




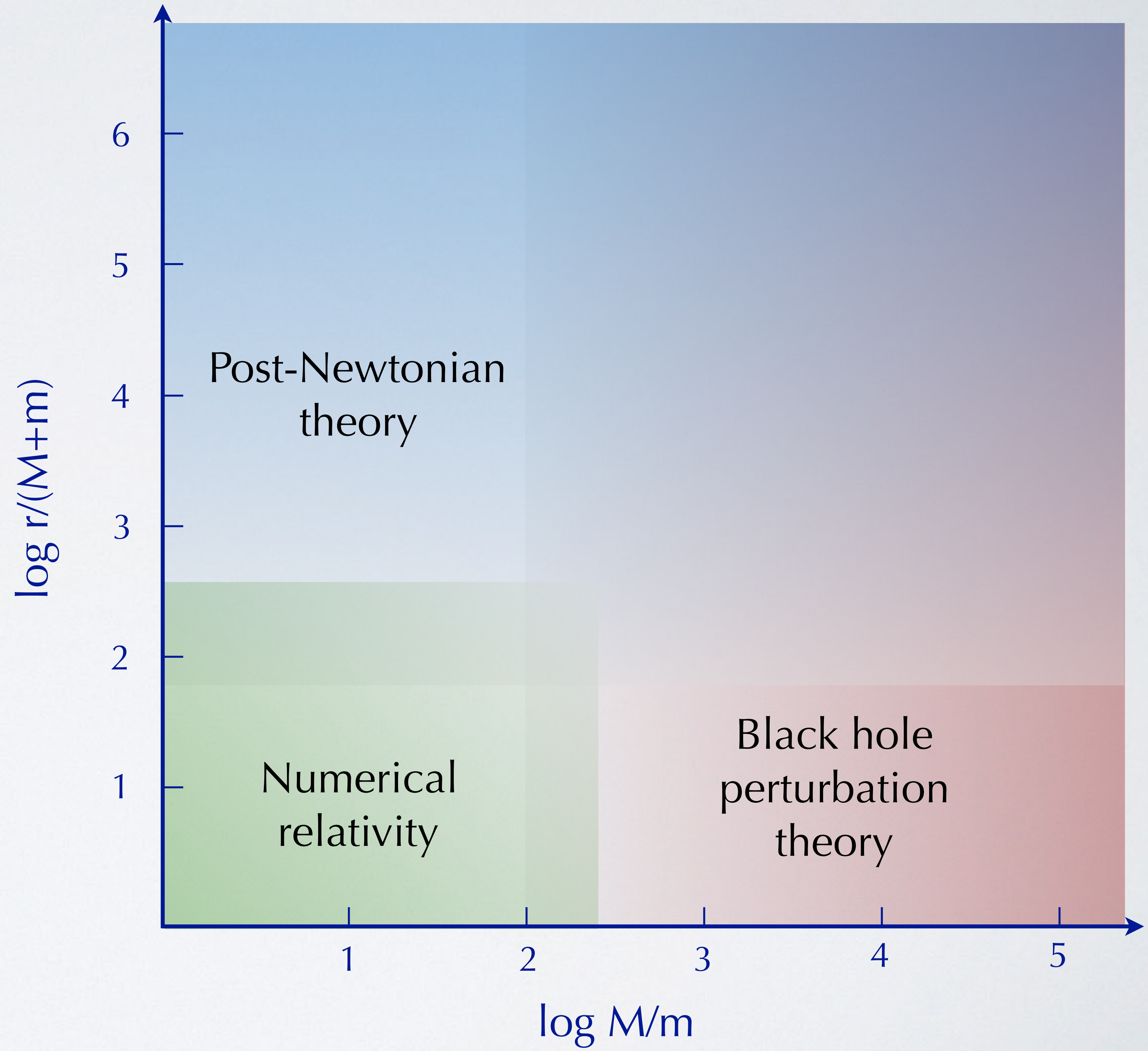
Thornburg, Wardell, 2017

Heffernan, et al., 2017

Jonathan Thornburg - Today

Peter Diener - Tuesday

There continues to be copious research in the PN + GSF overlap region



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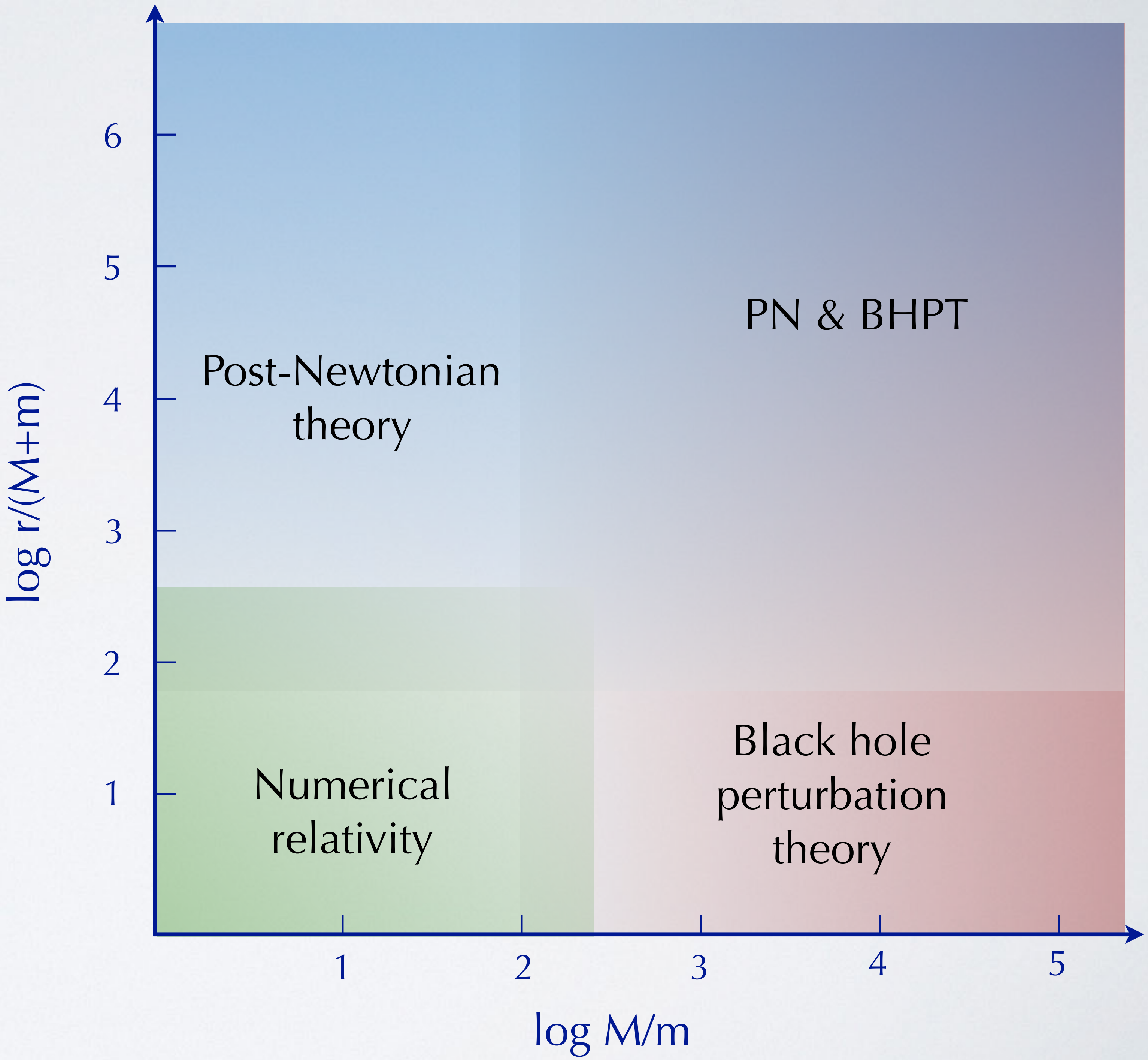

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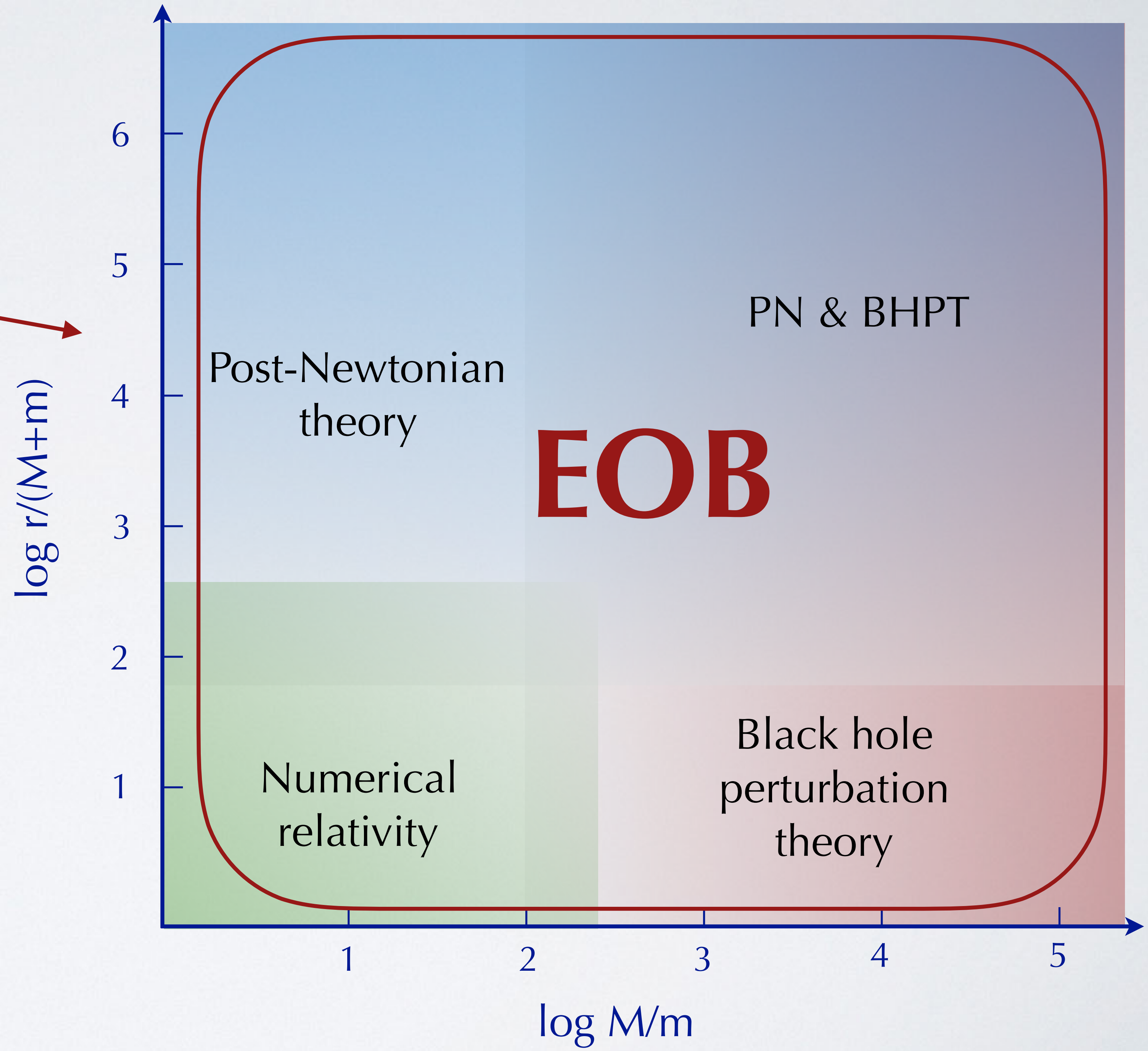
Chris Kavanagh - Tuesday



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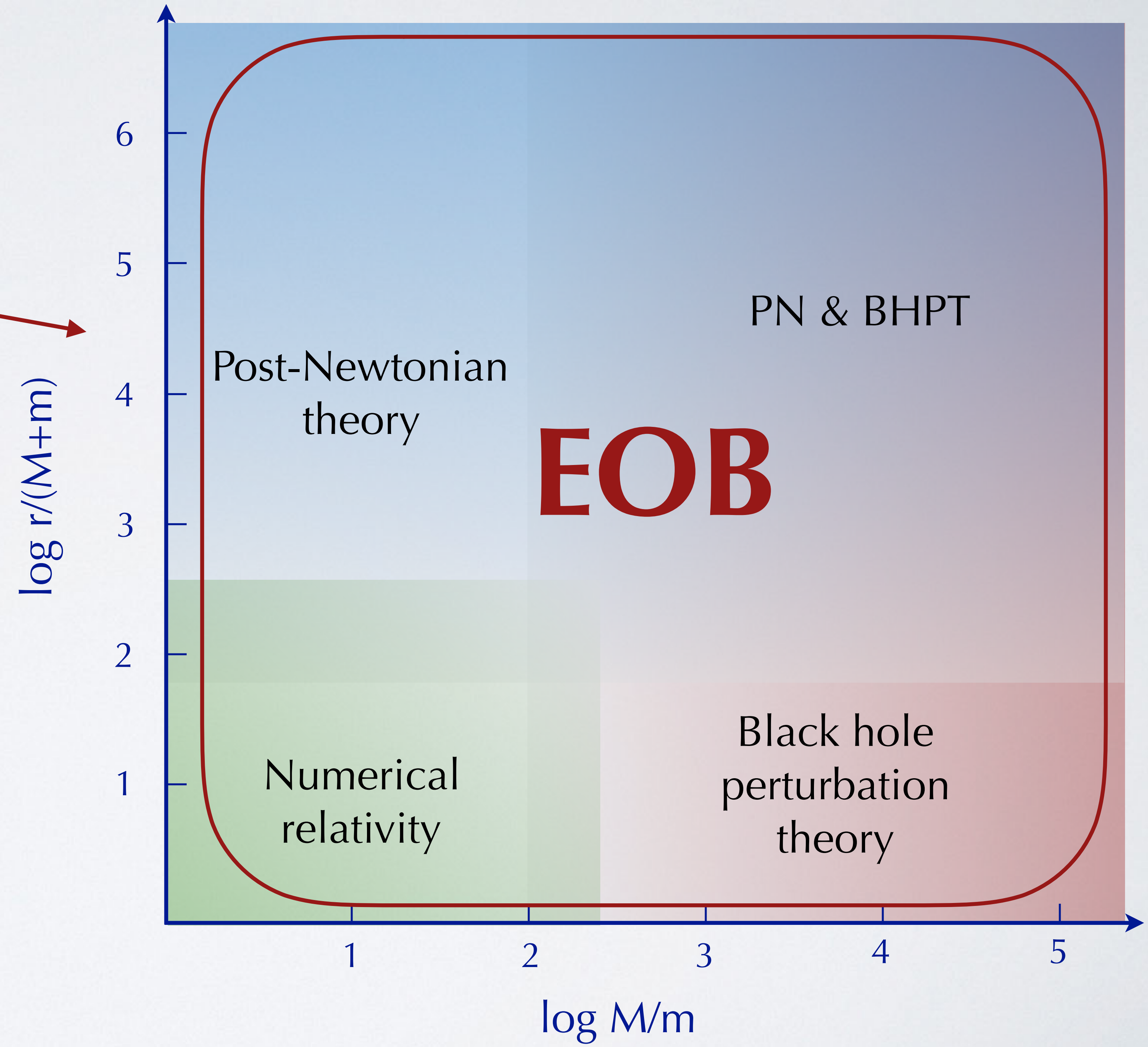
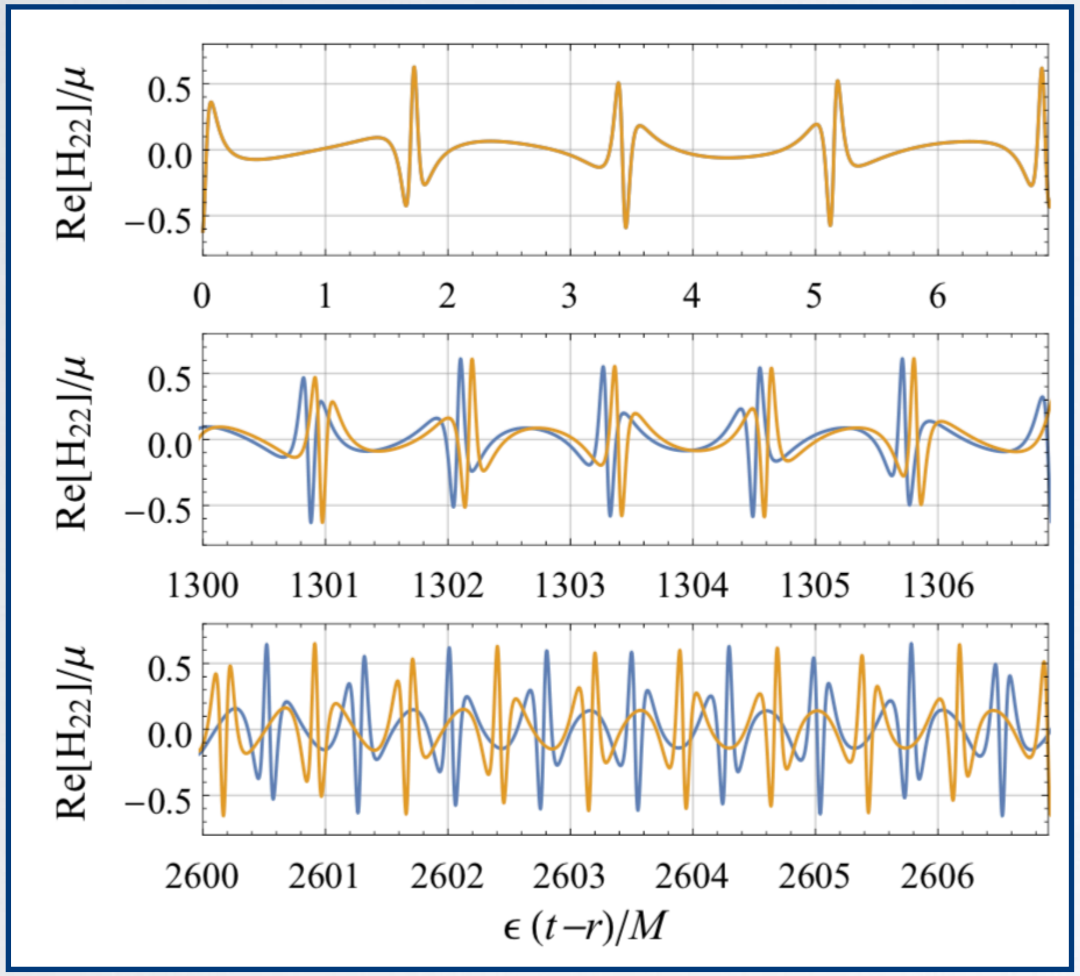
Chris Kavanagh - Tuesday

Andrea Antonelli - Tuesday



Inspirals with first-order self-force are becoming more common

Spinning secondary



Warburton, Osburn, Evans, 2017

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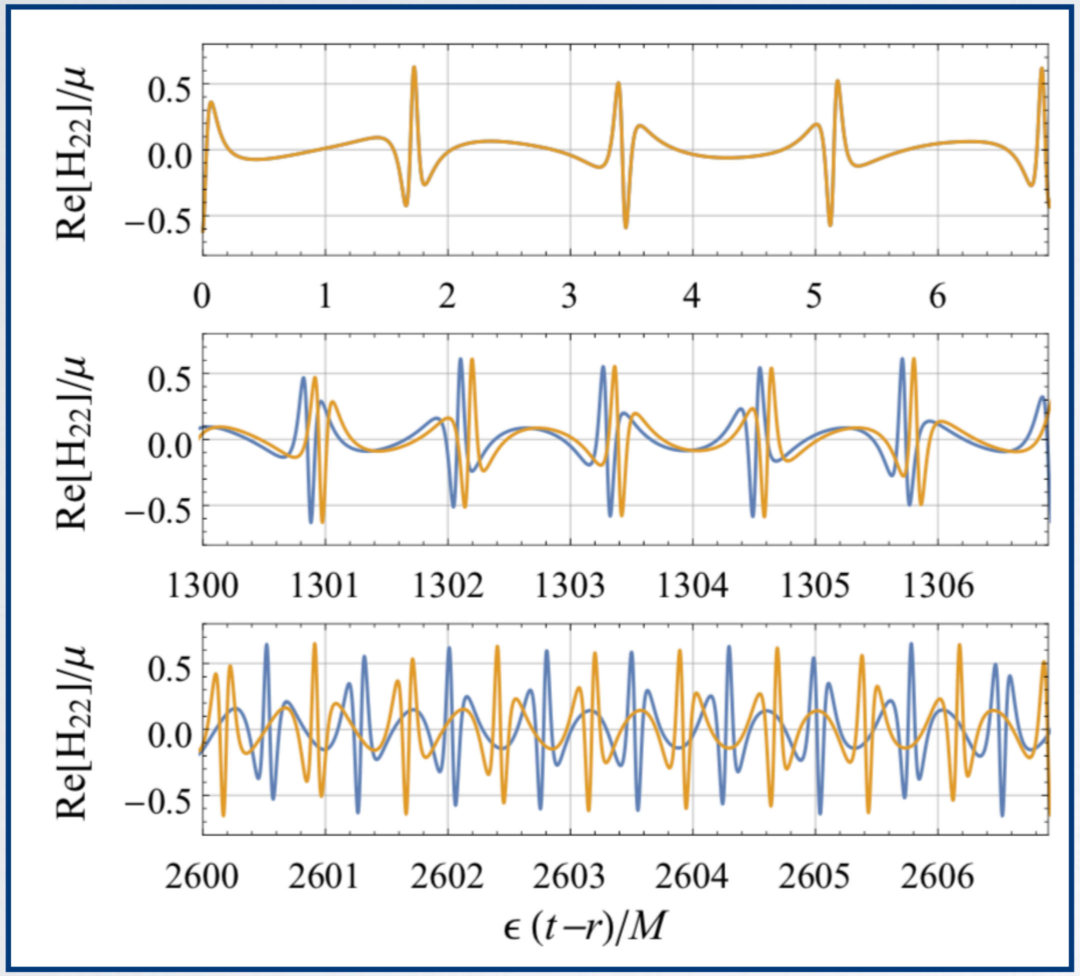

Warburton, Osburn, Evans, 2017

Sarp Akcay - Today

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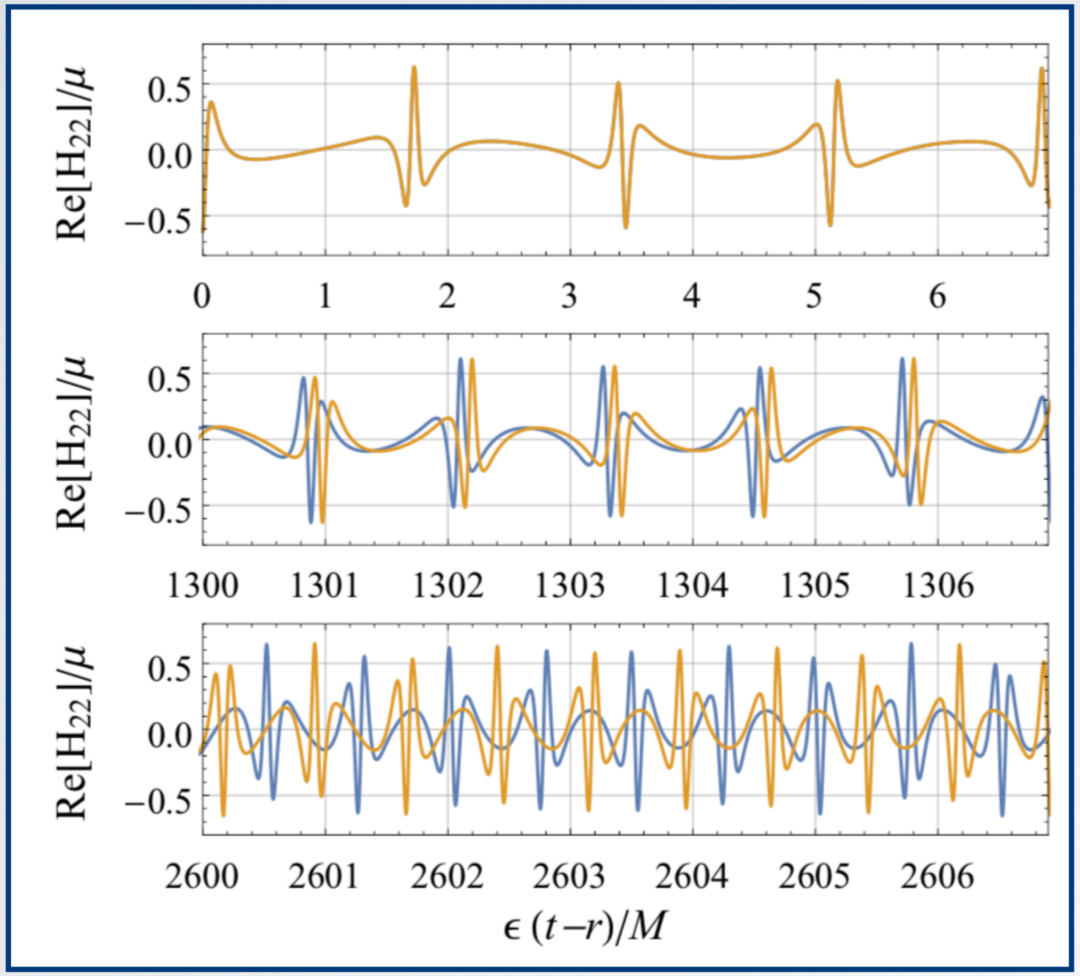

Warburton, Osburn, Evans, 2017

Sarp Akcay - Today

Thomas Osburn - Wednesday

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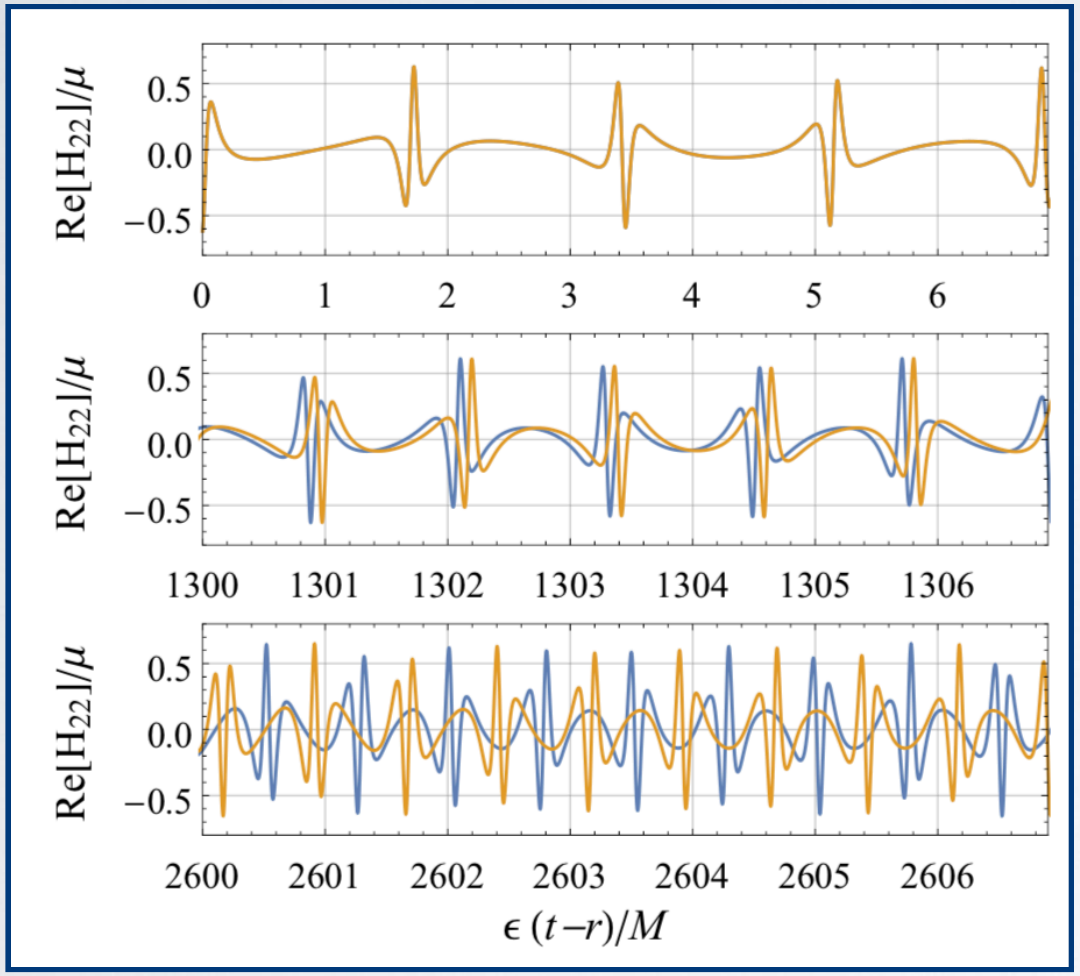
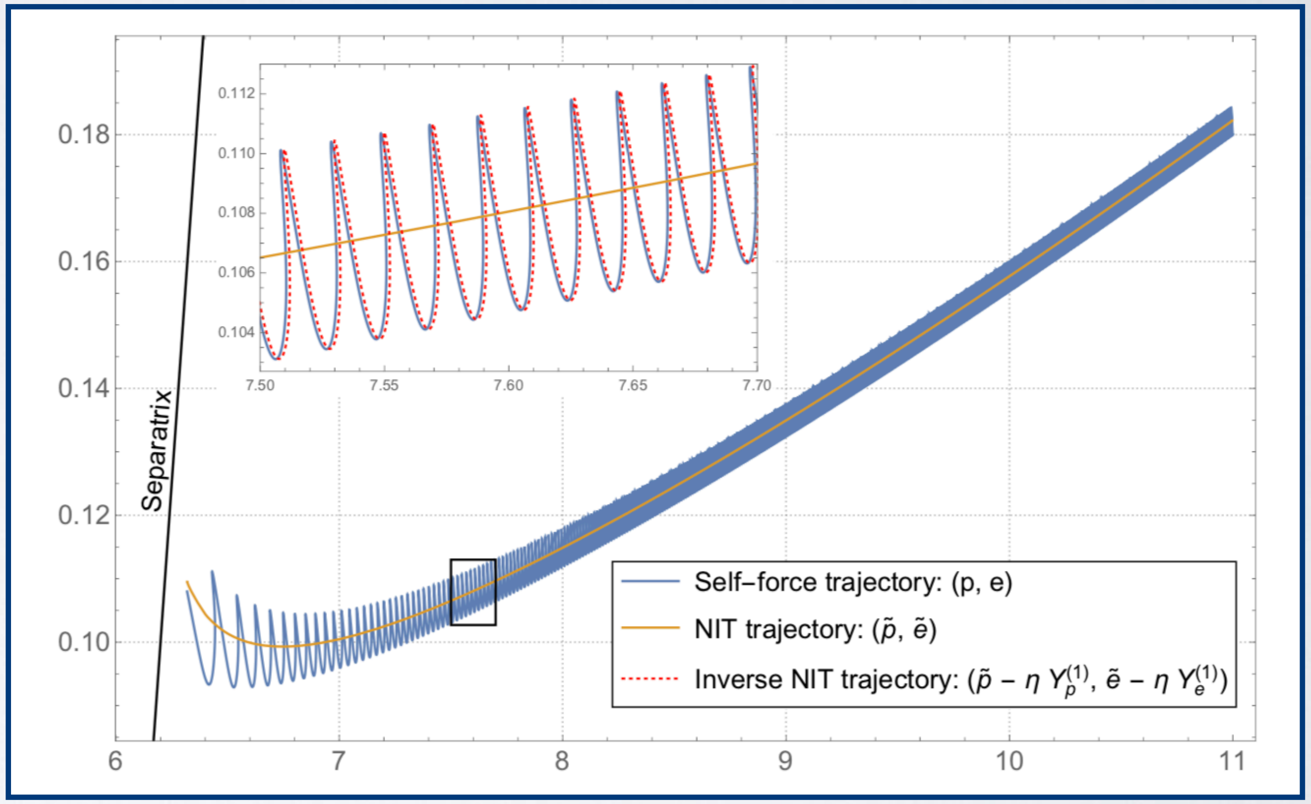


Warburton, Osburn, Evans, 2017

Fast inspirals



van de Meent, Warburton, 2018

Sarp Akcay - Today

Thomas Osburn - Wednesday

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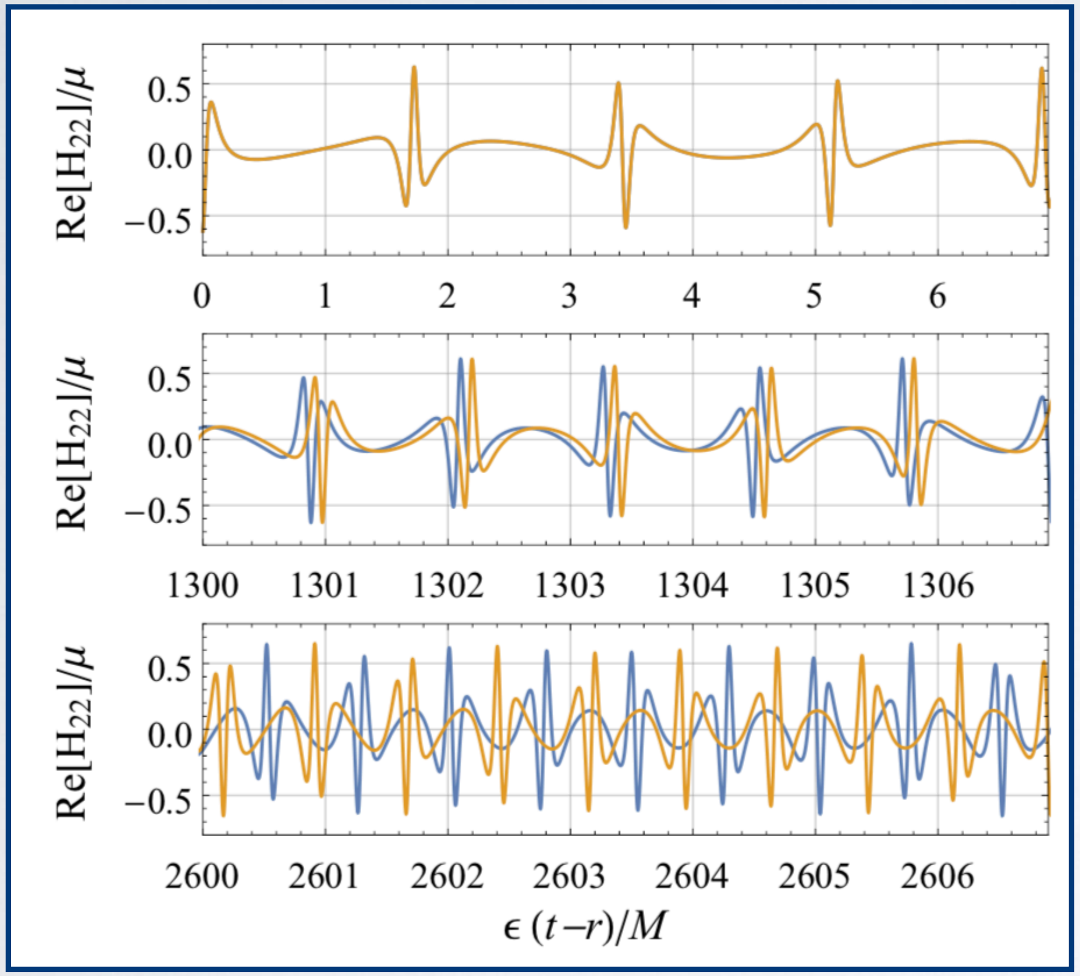

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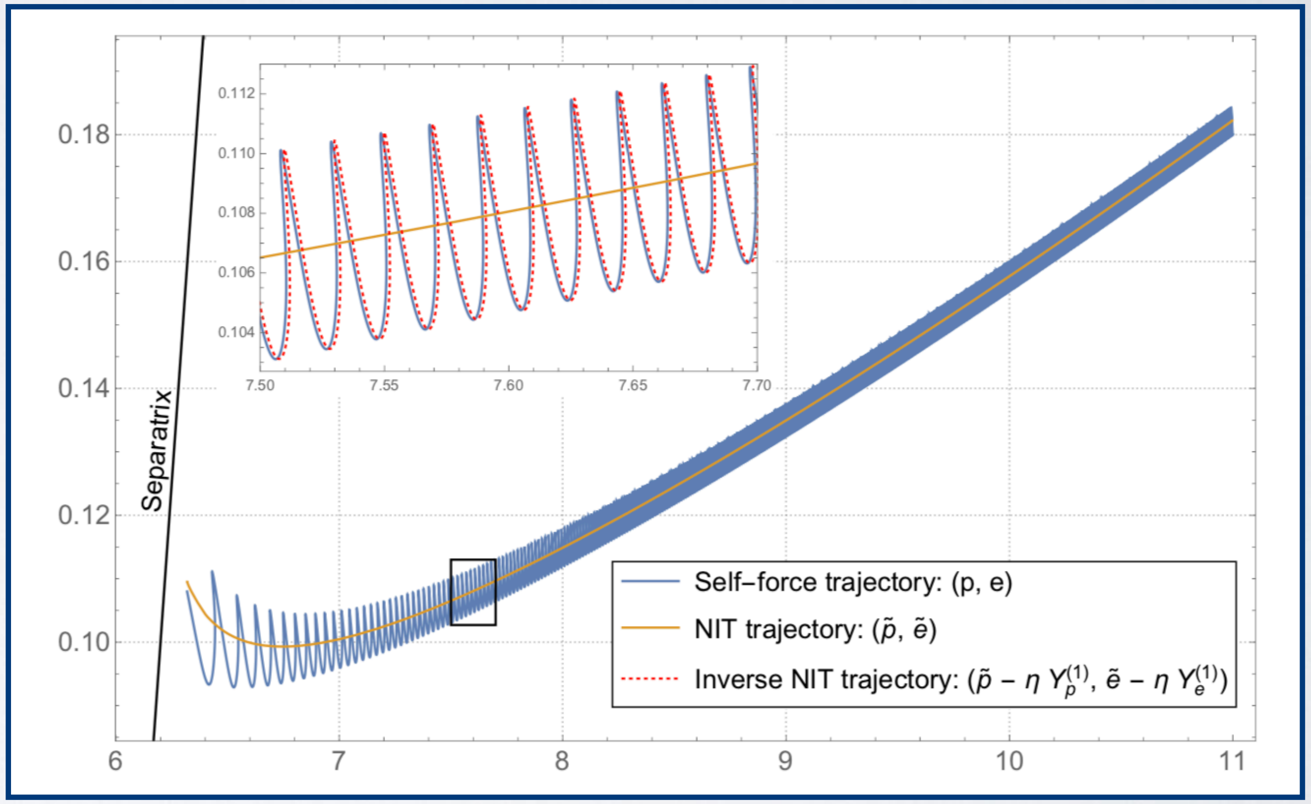

van de Meent, Warburton, 2018

Sarp Akcay - Today

Thomas Osburn - Wednesday

Niels Warburton - Wednesday

There's plenty of other self-force research that I don't understand yet

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Self-force in other dimensions:

Abraham Harte - Tuesday

Sumanta Chakraborty - Tuesday

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- Abraham Harte - Tuesday
- Sumanta Chakraborty - Tuesday

Second-order self-force:

- Adam Pound - Wednesday
- Barry Wardell - Wednesday
- Kei Yamada - Wednesday

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Self-force in other dimensions:

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Second-order self-force:

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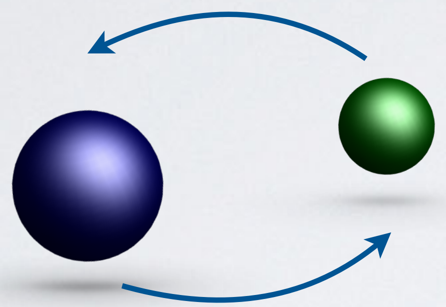
Kei Yamada - Wednesday

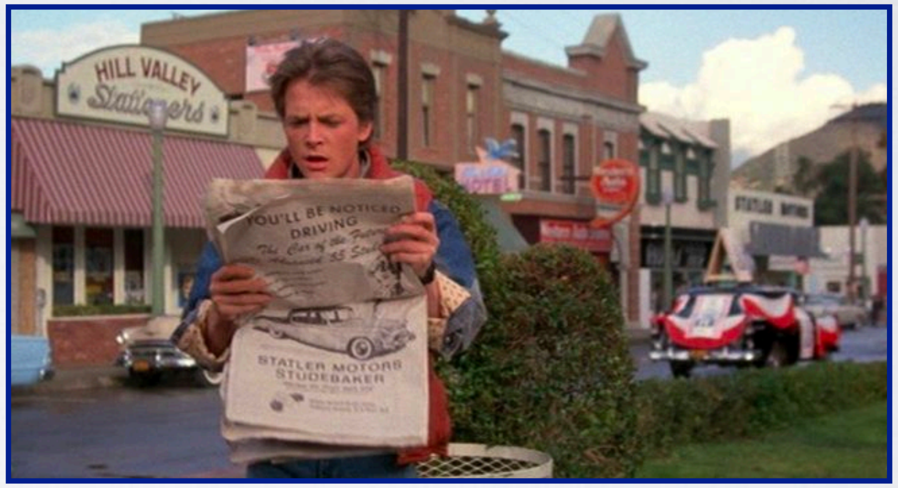
And more …

Outline

Why we're here



Yesterday



Today

Tomorrow

Outline

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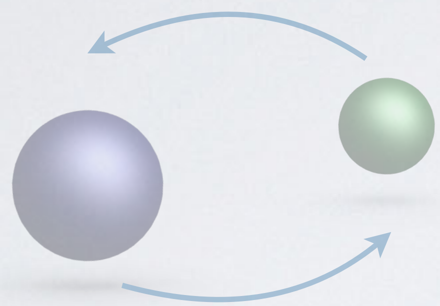

Yesterday

Today

Tomorrow

If we can't solve the first-order accurately enough, second-order will be unnecessary

$$\Phi_r = \kappa_0 \epsilon^{-1} + \kappa_{1/2} \epsilon^{-1/2} + \kappa_1 \epsilon^0$$

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Adiabatic, first-order, dissipative

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Resonances on Kerr

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+ First-order, conservative

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Adiabatic, first-order, dissipative

Oscillatory, first-order, dissipative

+ First-order, conservative

+ Adiabatic, second-order, dissipative

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Resonances on Kerr

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Adiabatic, first-order, dissipative

Oscillatory, first-order, dissipative

+ First-order, conservative

+ Adiabatic, second-order, dissipative

+ More terms if the particle has multipolar structure/spin

Is the time domain necessary?

Leo Stein:

Chuck Evans:

Is the time domain necessary?

Leo Stein:

“The universe exists in the time domain.”

Chuck Evans:

Is the time domain necessary?

Leo Stein:

"The universe exists in the time domain."

Chuck Evans:

"That's a very time-domain-centric point of view."

We should learn from numerical relativity

- Multiple codes + multiple techniques + multiple gauges are worth the time
- Pseudospectral/DG codes are worth the time
- We always need more people

Thank you!