Flatlanders never forget

Self-interaction in lower dimensions

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Capra 21
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21st Capra: Where are we?

Enormous progress on the self-force: Theoretical and computational methods, physical understanding, . . .

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- Most of this has been (ostensibly) motivated by astrophysics.
- But self-interaction occurs in many areas of physics; we can do more!

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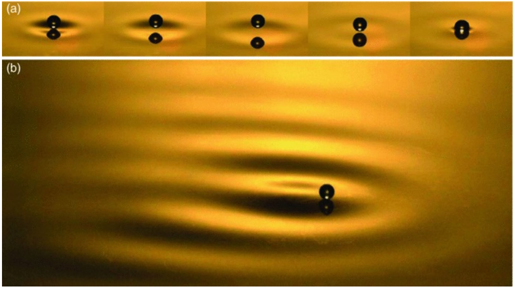
Electrons around magnetars

- $B \sim 10^{10}\,\mathrm{T}$
- Large $\dot{a}$ and (dipole energy) $>$ (rest mass)
- Even center of mass definitions break down!

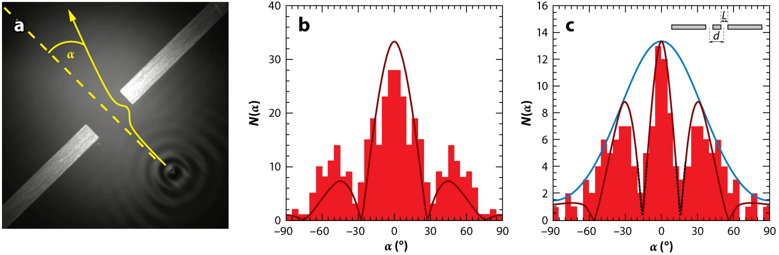
Different numbers of dimensions

- Many important condensed matter systems are effectively $2 + 1D$.
- Pilot wave hydrodynamics: An example of classical wave-particle duality [Couder, Fort, Bush, ...]



A tabletop $2 + 1D$ self-force experiment

Classical system with single-particle diffraction, tunneling, quantized orbits, Zeeman-type level splitting, and more!



Bush JWM. 2015.
Annu. Rev. Fluid Mech. 47:269–92

Is self-force similar when $d \neq 4$?

For even $d \geq 4$, yes; Detweiler-Whiting works.

Odd d is qualitatively different:

1. Huygens' principle strongly violated even in flat spacetime.
2. Tails are *unbounded*.
3. Detweiler-Whiting fails.

Wave propagation

In flat 4D spacetime, the retarded Green function is

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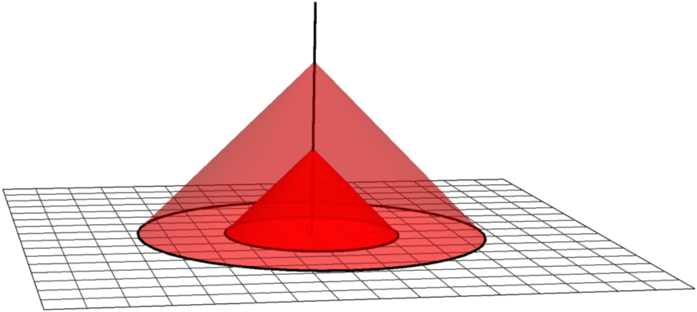
Dropping down one dimension,

$$G^{3D}_{\text{ret}} = \frac{\Theta_{\text{ret}}(-\sigma)}{(-\sigma)^{1/2}} \sim \frac{1}{t - t'}.$$

Perturbations are not sharp and travel *only* in timelike directions. Ripples on a pond...

Where does this come from?

Method of descent [Hadamard, 1923]: Let a line charge flash in and out of existence in $3+1$D and project into a plane.



Persistent memories

$$(d = 3 \text{ flat tail}) \sim t^{-1}, \qquad (d = 4 \text{ Schwarzschild tail}) \sim t^{-3}.$$

But $\int t^{-1} dt$ doesn't decay!

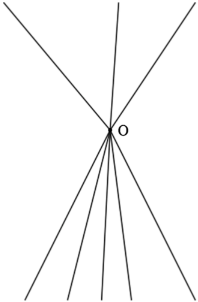
This *qualitatively* changes the importance of history dependence in self-force (and other) contexts.

A non self-force example: Memory effects

Charges scattering (and possibly annihilating) in $\mathbb{M}^3$ generically produce a late-time force

$$F \sim t^{-1/2}$$

on test particles [Satishchandran & Wald (2018)].



$\Rightarrow$ Velocity memory diverges when $d = 3$.

Simplest possible self-force

Isolated scalar charge which is created at $t = 0$ in $\mathbb{M}^3$:

$$\phi_{\rm ret} = \int q G_{\rm ret} d\tau \sim \int_0^t \frac{q_\infty d\tau}{[(t-\tau)^2 - r^2]^{1/2}}.$$

It may be shown that $\hat{\phi} \sim q_\infty \ln t$.

$\Rightarrow$ Charges "evaporate" logarithmically [Burko (2002)]:

$$m(t) - m(t') = q_\infty^2 \ln(t'/t).$$

Generic self-interaction

Considerable foundational and calculational developments are needed to go beyond this.

But self-force, self-torque, and extended-body effects now understood *non-perturbatively* [AIH, Taylor, Flanagan (2018)].

1. arbitrary d,
2. arbitrary g_{ab},
3. all multipole orders.

Focus here on monopole effects in $d = 3$ Minkowski...

Electromagnetic self-force in $\mathbb{M}^3$

With trivial boundary conditions and in a slow-motion limit,

$$F^i_{\rm self}(t) = -\frac{q^2}{2}\int_{-\infty}^{t} \dot{a}^i(\tau) \ln[(t-\tau)/\ell] d\tau.$$

- Depends on the past history of a particle's jerk $\dot{a}^i$.
- Weighting increases without bound in the distant past!!

$\mathcal{O}(qq_{ab})$ self-torques

Contribution to spin rate of change:

$$N^{ij}_{\text{self}} = 2q^{0[i}(F^{j]}_{\text{self}}/q)$$

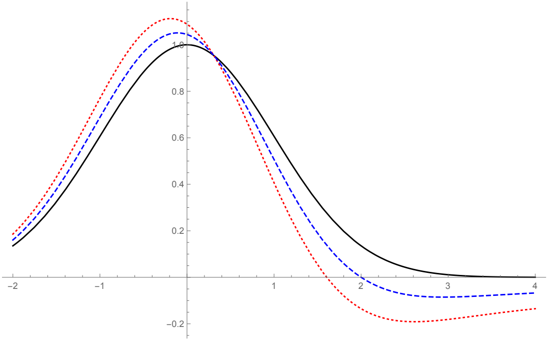
Contribution to hidden momentum:

$$N^{0i}_{\text{self}} = q^i{}_j(F^j_{\text{self}}/q).$$

Same log-weighted integrals of $\dot{a}^i$ for forces and torques...

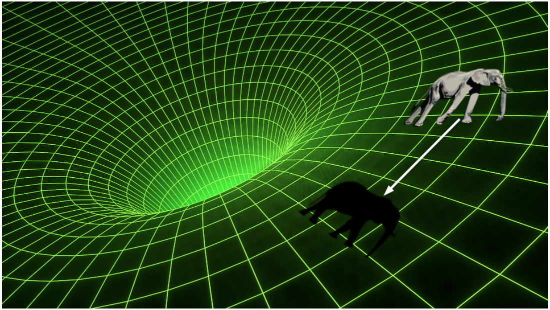
$\Rightarrow$ Solve *integral equations*, not differential equations.

Gaining some intuition: A Gaussian pulse



- Peak acceleration occurs before peak of applied force.
- This is causal; SF is sensitive to $\dot{a}$, not a.
- Persistent decaying deceleration at late times. . .

Generic kicks in $2+1D$



If a charge is initially inertial, is accelerated, and is then allowed to evolve freely, it tries to return to its initial state.

Aristotle strikes back

At late times, the velocity of kicked charge satisfies

$$v^i(t) = v_0^i + \frac{\Delta v^i}{1 + (q^2/2m)\ln[(t - t_0)/\ell]}.$$

Velocities asymptotically return to their initial values: $v^i \to v_0^i$

Bodies in $2+1D$ create their own "rest frames," which are never forgotten.

Heuristic explanation

Electric field due to the object's initial state acts like a decaying spring:

$$E^i_{\mathrm{init}} = -\frac{qx^i}{2t^2} + \mathcal{O}(|x|^3/t^4).$$

All solutions to

$$m\ddot{x}^i = qE^i_{\mathrm{init}}$$

vanish at late times.

Conclusions

- Self-interaction in lower dimensions is physically relevant.
- Slow field decay makes the self-force dominant even in simple systems.
- The shadow of a body's past creates its own rest frame in $2+1D$.

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Future questions

1. Can "quantum-like" behavior arise with nontrivial boundary conditions and/or extra particles? Explore numerically!
2. How can self-force ideas be applied in detail to real physical systems?