Flatlandiers never forget
Self-interaction in lower dimensions

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Work with Peter Taylor and Éanna Flanagan,
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Enormous progress on the self-force: Theoretical and computational methods, physical understanding, . . .

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Enormous progress on the self-force: Theoretical and computational methods, physical understanding, ...

- Most of this has been (ostensibly) motivated by astrophysics.
- But self-interaction occurs in many areas of physics; we can do more!
Many potential connections to experimentally-accessible systems...

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**Electrons around magnetars**
- $B \sim 10^{10} \text{ T}$
- Large $\dot{a}$ and (dipole energy) > (rest mass)
- Even center of mass definitions break down!
Many important condensed matter systems are effectively $2+1D$.

Pilot wave hydrodynamics: An example of classical wave-particle duality [Couder, Fort, Bush, ...]
A tabletop $2 + 1D$ self-force experiment

*Classical* system with single-particle diffraction, tunneling, quantized orbits, Zeeman-type level splitting, and more!

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Bush JWM. 2015.
Is self-force similar when $d \neq 4$?

For even $d \geq 4$, yes; Detweiler-Whiting works.

Odd $d$ is qualitatively different:

1. Huygens’ principle strongly violated even in flat spacetime.
2. Tails are *unbounded*.
3. Detweiler-Whiting fails.
In flat 4D spacetime, the retarded Green function is

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Perturbations are **sharp** and travel *only* in null directions.

Dropping down one dimension,

\[ G_{\text{ret}}^{3D} = \frac{\Theta_{\text{ret}}(-\sigma)}{(-\sigma)^{1/2}} \sim \frac{1}{t - t'}. \]

Perturbations are **not sharp** and travel *only* in timelike directions. Ripples on a pond...
Method of descent [Hadamard, 1923]: Let a line charge flash in and out of existence in $3+1$D and project into a plane.
Persistent memories

\[(d = 3 \text{ flat tail}) \sim t^{-1}, \quad (d = 4 \text{ Schwarzschild tail}) \sim t^{-3}.\]

But \( \int t^{-1} dt \) doesn’t decay!

This *qualitatively* changes the importance of history dependence in self-force (and other) contexts.
Charges scattering (and possibly annihilating) in $\mathbb{M}^3$ generically produce a late-time force

$$F \sim t^{-1/2}$$

on test particles [Satishchandran & Wald (2018)].

$\Rightarrow$ Velocity memory diverges when $d = 3$. 
**Isolated scalar charge** which is created at $t = 0$ in $\mathbb{M}^3$:

$$\phi_{\text{ret}} = \int qG_{\text{ret}} d\tau \sim \int_0^t \frac{q_\infty d\tau}{[(t - \tau)^2 - r^2]^{1/2}}.$$  

It may be shown that $\hat{\phi} \sim q_\infty \ln t$.

⇒ **Charges “evaporate” logarithmically** [Burko (2002)]:

$$m(t) - m(t') = q_\infty^2 \ln(t'/t).$$
Considerable foundational and calculational developments are needed to go beyond this.

But self-force, self-torque, and extended-body effects now understood non-perturbatively [AIH, Taylor, Flanagan (2018)].

1. arbitrary $d$,
2. arbitrary $g_{ab}$,
3. all multipole orders.

Focus here on monopole effects in $d = 3$ Minkowski...
With trivial boundary conditions and in a slow-motion limit,

\[ F_{\text{self}}^i(t) = -\frac{q^2}{2} \int_{-\infty}^{t} \dot{a}^i(\tau) \ln[(t - \tau)/\ell] d\tau. \]

- Depends on the past history of a particle’s jerk \( \dot{a}^i \).
- Weighting increases without bound in the distant past!!
\( O(qq_{ab}) \) self-torques

Contribution to spin rate of change:

\[
N_{\text{self}}^{ij} = 2q^0[i (F_{\text{self}}^j / q)
\]

Contribution to hidden momentum:

\[
N_{\text{self}}^{0i} = q^i j (F_{\text{self}}^j / q).
\]

Same log-weighted integrals of \( \dot{a^i} \) for forces and torques...

⇒ Solve integral equations, not differential equations.
Peak acceleration occurs before peak of applied force.
This is causal; SF is sensitive to $\dot{a}$, not $a$.
Persistent decaying deceleration at late times...
If a charge is initially inertial, is accelerated, and is then allowed to evolve freely, it **tries to return to its initial state**.
At late times, the velocity of kicked charge satisfies

$$v^i(t) = v^i_0 + \frac{\Delta v^i}{1 + (q^2/2m) \ln[(t - t_0)/\ell]}.$$ 

Velocities asymptotically return to their initial values: $v^i \to v^i_0$

Bodies in $2+1D$ create their own “rest frames,” which are never forgotten.
Electric field due to the object’s initial state acts like a *decaying spring*:

\[ E_{\text{init}}^i = -\frac{q x_i^i}{2t^2} + \mathcal{O}(|x|^3/t^4). \]

All solutions to

\[ m\ddot{x}^i = qE_{\text{init}}^i \]

vanish at late times.
Conclusions

- Self-interaction in lower dimensions is physically relevant.
- Slow field decay makes the self-force dominant even in simple systems.
- The shadow of a body’s past creates its own rest frame in $2 + 1D$. 
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Future questions

1. Can “quantum-like” behavior arise with nontrivial boundary conditions and/or extra particles? Explore numerically!
2. How can self-force ideas be applied in detail to real physical systems?