Progress in self-consistent evolution with a time domain scalar charge self-force code

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in collaboration with
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The problem.

We wish to determine the self-forced motion and field (e.g. energy and angular momentum fluxes) of a particle with scalar charge

$$\Box\psi^{\text{ret}} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau.$$

Two general approaches:

- Compute enough "geodesic"-based self-forces and then use these to drive the motion of the particle. (Post-processing, fast, accurate self-forces, relies on slow orbit evolution)
- Compute the "true" self-force while simultaneously driving the motion. (Potentially slow and expensive, potentially less accurate self-forces)

Effective source approach.

... is a general approach to self-force and self-consistent orbital evolution that doesn't use any delta functions.

Key ideas

- Compute a regular field, ψ^{R}, such that the self-force is

$$F_\alpha = \nabla_\alpha \psi^{\mathsf{R}}|_{x=z},$$

where $\psi^{\mathsf{R}} = \psi^{\mathsf{ret}} - \psi^{\mathsf{S}}$, and the Detweiler-Whiting singular field ψ^{S} can be approximated via local expansions: $\psi^{\mathsf{S}} = \tilde{\psi}^{\mathsf{S}}(x|z,u,a) + O(\epsilon^n)$.

- The effective source, S, for the field equation for ψ^{R} is regular at the particle location

$$\Box\psi^{\mathsf{R}} = \Box\psi^{\mathsf{ret}} - \Box\tilde{\psi}^{\mathsf{S}} = S(x|z,u,a,\dot{a},\ddot{a}),$$

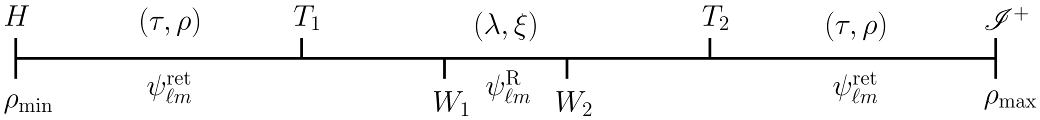
where $\Box\tilde{\psi}^{\mathsf{S}} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau - S$.

Self-consistent vs. geodesic evolutions.

- One main goal is to compare our self-consistent evolutions with Niels Warburton's geodesic evolutions.
- First attempt: 3+1D multi-patch finite difference code with a C^0 effective source.
- 3+1D accuracy limited by the non-smoothness of the source leading to high frequency noise with 2nd order convergent amplitude.
- Self-consistent evolutions agreed beautifully with geodesic evolutions within the errors (dominated by the noise).
- Next attempt: Improve the effective source smoothness to C^2.
- Geodesic evolution agreed with the C^0 evolutions and the frequency domain result with the noise reduced by more than an order of magnitude.
- However, we found differences between C^2 and C^0 results as soon as the back-reaction was turned on.
- 1+1D discontinuous Galerkin code without acceleration terms lost mode sum convergence when back-reaction was turned on.
- Now: 1+1D discontinuous Galerkin code with acceleration terms in the effective source.

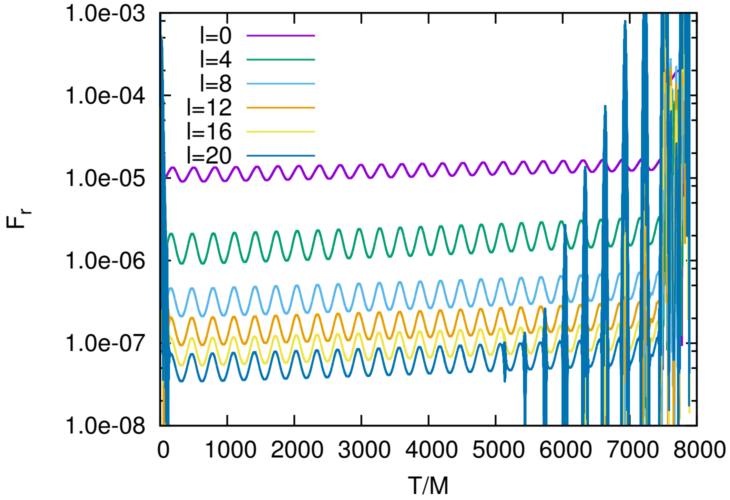
The code.

- Solves the spherical harmonic decomposed scalar wave equation in a Schwarzschild spacetime with a scalar effective source.
- Uses the Discontinuous Galerkin method for spatial discretization.
- Uses the method of lines approach and supports a number of time integrators.
- Uses a world-tube approach.
- Uses hyperboloidal slices, placing the computational domain boundaries at the horizon and $\mathscr{I}^+$.
- Uses a time dependent coordinate transformation to place the particle at a fixed coordinate location.
- The effective source include acceleration terms.
- Can read in frequency domain code initial data for small ℓ modes.



The state of self-consistent evolution at last Capra.

$p = 9.9$, $e = 0.1$, $q = 1/8$. Only four-acceleration passed in to the effective source!!!!



What was wrong?

- Noticed that a bit of noise appeared in the extracted self-force shortly after each periapsis passage.
- Noticed that the same thing happened after each apapsis passage.
- Turns out it was caused by the calculation of u^r.

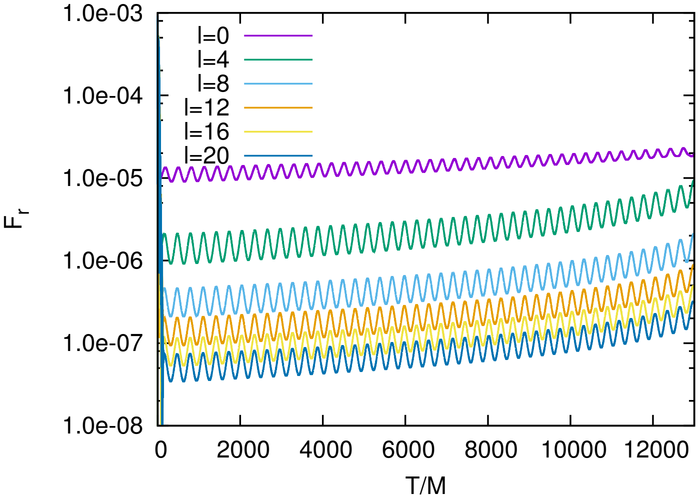
$$u^r = \pm\sqrt{E^2 - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{L_z^2}{r^2}\right)},$$

$$E^2 = \frac{(p-2-2e)(p-2+2e)}{p(p-3-e^2)} \text{ and } L_z^2 = \frac{p^2M^2}{p-3-e^2}$$

- Instead of $u^r \approx 10^{-16}$ we got $u^r \approx 10^{-8}$ just before and after peri- and apapsis.
- This apparently generates enough noise to trigger a feedback instability when the back-reaction is applied.
- Easy fix: do this calculation in quad precision.

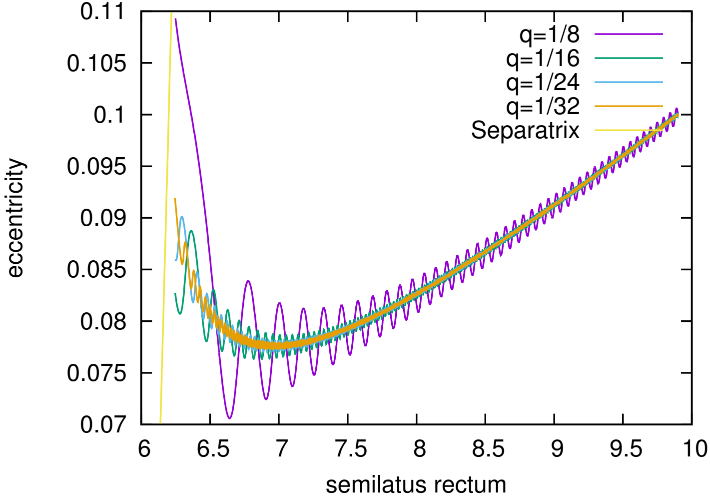
The state of self-consistent evolution now.

$p = 9.9$, $e = 0.1$, $q = 1/8$. Still only four-acceleration passed in to the effective source.

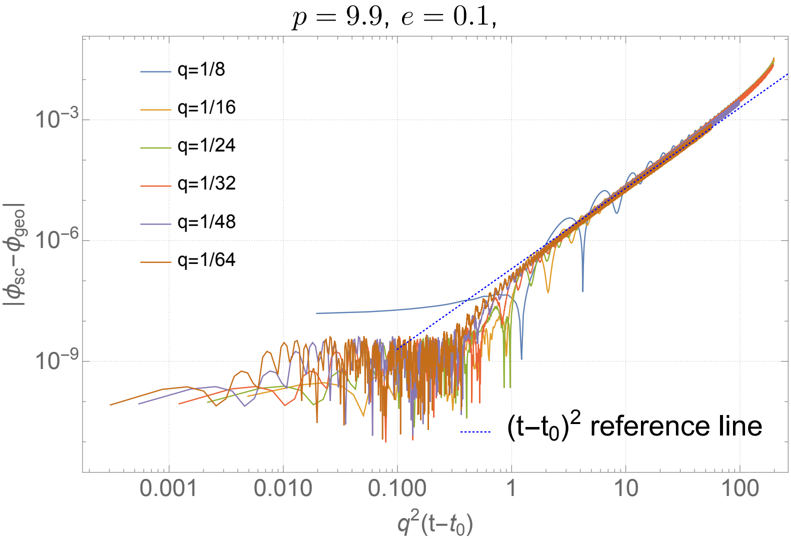


The state of self-consistent evolution now.

$p = 9.9,\ e = 0.1.$



The state of self-consistent evolution now.

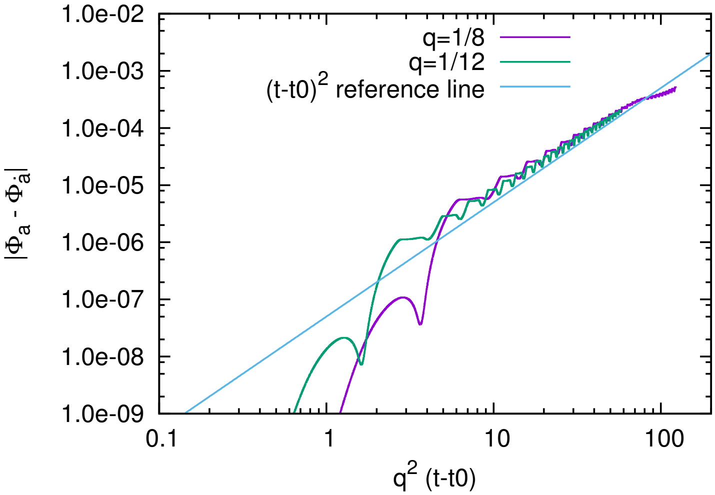


The state of self-consistent evolution now.

- We still have instabilities when we pass in $\dot{a}$ and/or $\ddot{a}$.
- At first we thought this was caused by extra noise in the numerical calculation of time derivatives of the 4-acceleration.
- We experimented with smoothing finite differences (made complicated due to time varying Δt): Did not help much.
- We then implemented an Adams-Bashford-Moulton multi-value (ABMV) time integrator where higher time derivatives of the variables are part of the evolution system: Extended the run time but still instabilities.
- Question is: How important are the higher derivatives of a?
- It turns out that the ABMV scheme does allow for long evolutions if $\ell_{\max}$ is not too large. Comparing the phase evolution between runs with and without $\dot{a}$ terms may help quantify this.

The state of self-consistent evolution now.

$p = 9.9$, $e = 0.1$ (runs by Aaron Hodson, ongoing).



Conclusions and Outlook.

- We can now do self-consistent orbits where the effective source depends on the acceleration.
- Preliminary results consistent with the expectation that the phase error between 'geodesic' and 'self-consistent' evolutions grows as $(t - t_0)^2$ and scales as q^2.
- Need to understand the numerical errors better before we can say anything definite about the magnitude of the phase error.
- Need to finish investigation into importance of time derivatives of the acceleration. REU student Aaron Hodson is working on this.
- Gravitational perturbation codes (Lorenz, Regge-Wheeler-Zerilli and Teukolsky) are in various stages of development/testing.
- Currently undertaking a redesign and rewrite of the code. Once this is done, we plan to release the code as open software (http://bhptoolkit.org?).
- Plan to extend the code to handle Kerr as well.

Teaser: New code for Teukolsky in Schwarzschild by Sarah Skinner

$s = -1$, $\ell = 1$, DG-order=16.